

RISK OF PRIVATE EQUITY FUND-OF-FUND INVESTMENTS - A DETAILED CASH FLOW-BASED APPROACH

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Fund-of-fund investments in private equity are becoming increasingly popular due to their potential to diversify across several private equity funds. However, due to the illiquidity and opaqueness of private equity markets only limited knowledge is available on the actual risk profile of fund-of-funds. Hence, we aim at contributing to the field by modeling, quantifying, and testing sensitivities of the risks associated with such investments. We design a detailed cash flow model and adopt the value-at-risk approach, which is modified to fit the two most common measures of return, namely internal rate of return (IRR) and multiple. Based on a proprietary data set of historical transactions we simulate fund-of-funds investments. Further, sensitivity analyses of central input parameters show the impact on risk and return. While reducing the management fee and the carried interest would lead to higher returns, increased diversification reduces risk. Based on our model, reducing the current industry standard of 2% management fee and 20% carried interest of private equity funds increases the mean IRR by 4.3 percentage points.

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1. Introduction

In recent years, private equity as an asset class has gained more and more importance. Investors like wealthy individuals, pension funds, hedge funds, fund-of funds (FoF), etc. sought investment opportunities in this asset class, leading to a record amount in committed capital in 2006 and 2007, both in nominal terms and as a fraction of the overall market (see [10]). The business model of private equity investors aims at increasing the value of target companies through financial, governance and operational engineering, increasing the competitiveness of portfolio companies and the domestic economy (see [11]).

Given the relative opaqueness of the private equity market, not only is information difficult to obtain but scholarly work is scarce in comparison to public equity markets. Hence it is barely surprising that while private equity scholars and practitioners have come to a certain degree of agreement concerning the measurement of returns, the problem of assessing its risk is not yet sufficiently solved.

The risk of investments in FoF, which collect money to invest in a variety of private equity funds (PEF), is particularly underresearched. According to the European Private Equity & Venture Capital Association (EVCA), FoF currently account for approximately 15% of total funds raised in the private equity market, which equals approximately 12 billion Euro in 2007. Given this high importance of FoF, it is essential to assess their riskiness in order to create comparability of asset classes and enable investors to make informed decisions about investments in this increasingly important asset class.

For measuring the return of a PEF or FoF there exist uncontroversial indicators, above all the internal rate of return (IRR) and the money multiple. No such agreement exists among scientists or practitioners concerning the assessment of risk. This is noteworthy since, particularly for private equity FoF in Europe, the issue of risk measurement is especially important: Within the European Union banking regulation has been subject to several changes under Basel I and II. Among others, these changes increased the banks' opportunities to introduce own measures of risk to assess the corresponding capital requirements on their balance sheets. Therefore, the application of appropriate methods, which exceed the simplifying standard approaches in accuracy, are rewarded by lower capital requirements. This enhances the banks' possibilities to use debt and hence increase the leverage effect for its investments. Belonging to the customer base of FoF, banks increasingly demand the assessment of the risk-return profile of their private equity FoF investments.

From a scientific point-of-view, the problems associated with risk measurement of private equity investments have been acknowledged for a long time. First, an investor in a PEF is not able to assess the values of the portfolio companies throughout their lifetimes because any reported value does not necessarily match the 'real' value, i.e. the value realized when a portfolio company is sold. If the investment of the PEF is not yet realized, the private equity firm estimates the net asset value on a fair value basis, which is then reported to the limited partners, e.g. FoF investors.^a Second, estimating the risk of private equity investments from historical

^aThe International Private Equity and Venture Capital Valuation Guidelines (IPEV) defines fair value as an indication of the interim performance of a portfolio company. It should be viewed as an estimate based on specific characteristics of the security, as well as on market forces. Fair value accounting was again stressed by the IPEV Board on 20th November 2008. For further information on valuation guidelines see <http://www.privateequityvaluation.com/#Board>

investments suffers from the relatively short history of this asset class and the historical lack of accounting and disclosure requirements. Third, common risk measures, e.g. the standard deviation in combination with the mean value, do not sufficiently take into account the high risk of great losses that are characteristic of alternative investments, like private equity and hedge funds.

Considering these characteristics of private equity investments, we analyze the risk-return-profile of private equity FoF investments applying the idea of value-at-risk (VaR). The concept of VaR is not only capable of accounting for the prevalent downside risk, but is also an appropriate indicator to capture the long-term investment horizon and the difficult valuation during the holding period ([3]).

Our paper makes three major contributions: First, we design a model which allows FoF investors to assess the risk, not only of the investment class, but also for individual FoF. It extends the risk approach of VaR to the common performance measures in private equity, namely IRR and multiple. Given that the predominantly positive performance of private equity FoF in the past, the VaR concept is extended to fit these two common performance measures. These modifications permit to quantify changes in non-negative values. Further, our model is unique in that it replicates all levels of cash flows and considers all major risks of the investment in a FoF. Second, we empirically test our model and quantify the VaR based on a proprietary data sample of a German FoF. We are able to make use of historical data on relevant fund characteristics such as carried interest, management fee and hurdle rate. Third, we investigate the sensitivity of the VaR to variations in key parameters. Our proprietary sample was acquired through a cooperation with a private equity FoF, which has access to fund-related data during the due diligence process. It includes historical cash flows of portfolio companies of PEF. Even though the sample is quite large, Monte-Carlo simulation will be used to artificially create new FoF for two reasons: The first is that one FoF invests in a huge number of portfolio companies through the allocation of capital to multiple PEF. Therefore, the number of FoF that resulted from the sample is therefore not sufficiently large for a proper analysis. The second reason, which is especially decisive under current market conditions, is that, by creating new funds, it is also possible to simulate new market conditions, allowing the model not only to analyze the historical development of private equity FoF investments, but also to make more reliable projections of possible future developments.

The remainder of the paper is structured as follows: Section 2 describes the risks associated with an investment in a private equity FoF. Section 3 illustrates the basic functioning of the simulation approach used throughout the study. Section 4, then, gives a deeper insight into the integration of the identified risk categories into the simulation process. Part 5 deals with the actual simulation procedures and results, while part 6 concludes the article.

2. Private Equity Investing

In the broader sense, private equity describes investments in non-listed companies. private equity in the narrow sense, subject of the present paper, predominantly refers to investments in buyouts of established and mature companies. Including venture capital in the analysis, as an additional type of private equity in a broader sense, would mix up the characteristics of two significantly different asset classes, particularly in terms of prevalent risks (see [9]), and hence some of the crucial

information would become blurred.

Basically, investments in private equity are done in three different ways: (1) The direct investment bears the highest risk but also offers the potentially highest return. In this case, the investor buys a share in the company directly from the owner and, in that way, also shares the entire potential for loss and gain. (2) A private equity fund is a less risky means of investing in private equity. The fund manager collects the capital from one or several investors and then distributes it in investments in several portfolio companies. This reduces the investor's risk through diversification effects, but also reduces the return by the amount needed to compensate the management of the PEF. Diversification benefits can be amplified by investing (3) in a FoF. This investment structure adds another layer of diversification - but also fees - by collecting capital from investors and investing the money in a variety of selected private equity funds. Obviously, the concept of a FoF exists for many types of asset classes. In this context, however, the term FoF will be used to describe a FoF investing solely in PEF. In modeling the risk of private equity we will draw on this most complex investment structure in order to consider as many relevant risk issues as possible. To our knowledge, this is the first attempt to scientifically model and compute the risks for investors in private equity FoF with an appropriate risk indicator.

The cash flows associated with private equity FoF investments are highly complex. However, before any capital is committed, the investments have to be identified according to the following procedure: In the screening phase, investors evaluate potential FoF matching their investment objectives. Once the preferred FoF is identified, an investment contract is signed that contains the specifications and characteristics of the fund (e.g. the term, management fee, investment focus, etc.) as well as the investor's desired investment amount. In the next step, the FoF runs an investment process similar to the one of the investor that aims at identifying and investing in PEF. Then, PEF try to find and invest in promising portfolio companies (PCs). For the purpose of clarity, we explain this process in a simplified way, while in reality, which is reflected in our model, investors as well as FoF typically invest in more than one FoF or PEF respectively.

Most of the cash flows in this investment chain - i.e. investor, FoF, PEF - commonly take place in tranches only. The size of each tranche depends on the specification in the investment contract. In detail, the flow of capital can be described as follows (numbers and letters in parentheses correspond to Figure 1):

The investment process is explained for the simplified construct of one investor, one FoF, one PEF and one PC. First, the called amount is paid by the investor to the FoF to cover the called capital from the private equity fund (1). The FoF then distributes the called capital to the PEF, (2), which in turn, buys shares in a PC (3). After a certain holding period the PEF sells the PC and receives the cash flow from the selling price of the equity stake (4). This cash flow is then transferred to the FoF (5) and the FoF redistributes the cash flow to the investor (6).

In addition to these investing and divesting cash flows, a part of the capital that is committed to a FoF or PEF is needed to compensate the fund management for its efforts. Although the common amounts may be different, the general structure is the same for PEF and FoF. These charges are composed of the management fee (M_{FoF} and M_{PEF}) and the so-called carried interest (C_{FoF} and C_{PEF}). The management fee is due during the whole lifetime of a fund and is meant to reimburse

the fund management for its expenditures. The management fee usually amounts to circa 2.0% of the committed capital per year for a PEF, M_{PEF} (see [5]) and to circa 0.5% for a FoF, M_{FoF} (see [22]).^b This full amount is only applicable during the investment period, which comprises the first years of a fund's lifetime that is scheduled for the time-consuming investment activity. After this investment period, the management fee usually applies to the actual invested capital. The goal of the carried interest is to align the interests of investors and fund management. Therefore, once a fund has paid back all the money invested plus a certain minimum rate of return, the so-called hurdle-rate amounting to 6 to 8% (see [19]), the fund management participates in all future profits with the carried interest. Fund managers hereby typically receive 20% on the PEF level, C_{PEF} , (see [5]) and 5 to 10% on the FoF level, C_{FoF} (see [22]).^c

Besides the fee structure, cash flows within the FoF and the PEF occur that are related to interest accumulation. Calling capital and redistributing funds in tranches, either by FoF or PEF, decreases the transaction costs associated with every capital call and redistribution. The remainder of the capital is kept in an interest-bearing account at common market interest rates leading to additional cash

^bThese numbers will be empirically tested and simulated in our analysis.

^cSee footnote b.

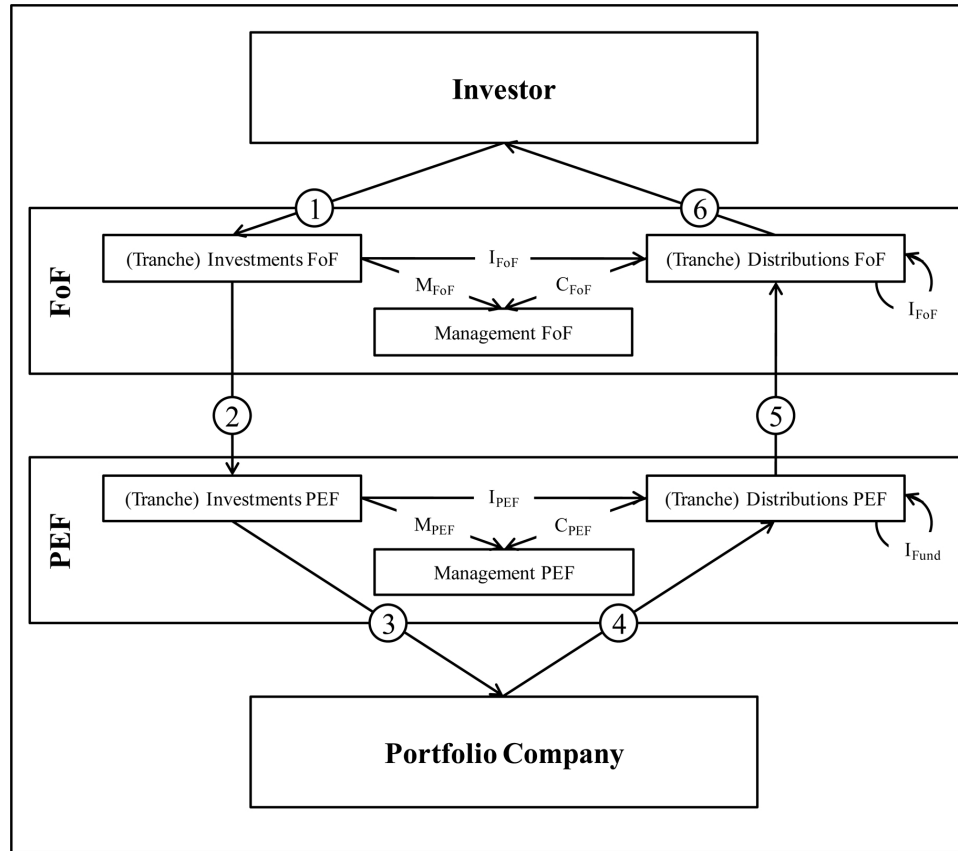


Fig. 1. Cash flows associated with an investment in a private equity fund-of-funds.

flows to the investor (I_{FoF} and I_{PEF}).

As private equity investments historically used to be traded on secondary markets only occasionally, return measurement usually relies on the cash flow history of a fund. In order to measure the rate of return of a PEF or FoF, a variety of measures are proposed and used in the academic as well as practical world. Multiple and IRR are the most important measures to assess the performance of PEF and FoF. Academics as well as practitioners have adopted these two as the standard performance measures for PEF and FoF.

The IRR is the discount rate that yields a net present value of zero for the cash flow history of an investment:

$$0 = \sum_{t=0}^T \left(\frac{\sum_{i=1}^I cf_{i,t}}{(1 + IRR)^t} \right). \quad (2.1)$$

Three shortcomings restrict the use of IRR (see [17]). First, for one cash flow stream there are as many mathematical IRR solutions as the cash flow has sign changes. Second, under some circumstances the IRR is incalculable. Third, the IRR implicitly assumes that interim cash flows are (re)invested at the same IRR. In most cases this reinvestment assumption is unrealistic, especially for private equity investments, as neither PEF nor FoF are immediately able to reinvest in projects with the same IRR. While the first two restrictions do not limit our analysis, the third restriction limits the comparability to other asset classes. However, as IRR is used as the standard measure for the performance of investments in PCs and also for investments in PEF and FoF, we apply this measure to our analysis.

Next to the IRR, academics as well as practitioners use a multiple approach to calculate the performance for private equity investments. As this paper analyses realized investments, the most relevant type of multiple for investors is a modified form of the Distributed to Invested Capital (mDIC) multiple.^d If this multiple is greater than zero, the investor received more cash flows from his investment than she paid in. The calculation of the mDIC looks as follows

$$mDIC_{cf,x}^T = \left(\frac{\sum_{t=0}^T \sum_{k=1}^K cf_+^{kt} \cdot x_+^{kt}}{-\sum_{t=0}^T \sum_{k=1}^K cf_-^{kt} \cdot x_-^{kt}} \right) - 1, \quad (2.2)$$

with '+' indicating positive cash flows and '-' standing for negative cash flows. Cash flows are not discounted or modified. For both types of performance measure we use the actual cash flows, net of fees and interest, for investors of FoF.

In terms of measuring the risk of private equity, the concept of VaR has recently attracted increasing attention. After JP Morgan had developed the state-of-the-art calculation in the early 1990s, the concept eventually became accepted by the general finance community when the Basel Committee on Banking Supervision allowed financial institutions to quantify their market risks with internal VaR models (see [21]). As a consequence of this increased use by investors, more and more PEF and FoF must report their risk positions as VaR in order to meet their customers' requirements. In addition, VaR simply seems to be a good choice of a risk measure in private equity since other, traditional risk measures used in public markets are

^dFor alternative multiples see [2].

difficult to apply to private equity, given the long-term investment horizon and the generally low availability of market prices during the holding period (see [3]).

In general terms, an investment's VaR is defined as the largest loss likely to be suffered over a given holding period at a given confidence level, i.e. with a given probability. Mathematically, value-at-risk is the α -quintile of the investment distribution:

$$VaR_{(1-\alpha)} = -F_{cf,x}^{-1}(\alpha) = -mDIC_{cf,x}^{-1}(\alpha). \quad (2.3)$$

The α -quintile is well-defined by the generalized inverse distribution function

$$F_{cf,x}^{-1}(\alpha) = \sup \{y \mid F_{cf,x}(y) \leq \alpha\}. \quad (2.4)$$

Given the two common performance measures in private equity FoF, the IRR and the mDIC, and their often non-negative distribution, we additionally introduce modified risk measures: the internal Return at Risk (iRaR) and the Multiple at Risk (MaR). According to the concept of VaR, the iRaR (MaR) is the lowest return (multiple) likely to be achieved over a given holding period at a given confidence level. In comparison to the concept of the VaR, these measures are able to analyze the changes in the risk of a FoF in asset classes that historically tend to have non-negative performance profiles.

In order to calculate any of these measures, we need $F_{CF,X}$ as the respective distribution function of profits and losses (of IRR or multiples) of the risky investment in a given holding period $[0, T]$. In the present case, this stochastic distribution results from the random occurrences of the p cash flow streams in any period $0 \leq t \leq T$.

$$cf_t = (cf_{1t} \cdots cf_{Pt}). \quad (2.5)$$

These cash flows, which will be introduced in more detail in Chapter 4.1, are subject to several risks. The set of all possible future values of the m relevant risks in the model is given by vector X . A random occurrence of X describes one possible future model state of the risk factors in t

$$x_t = \begin{pmatrix} x_{1t} \\ \vdots \\ x_{Mt} \end{pmatrix}. \quad (2.6)$$

However, each cash flow is exposed to a time-constant subset of these risks only. Hence the overall risk of cash flow p in t is given by x_{kt} for which holds

$$x_{kt} \subseteq x_t. \quad (2.7)$$

Hence the investment's value at T is given by a function of the p cash flows in period $[0, T]$ subject to a p -specific set of risks:

$$F_{cf,x}^T = \sum_{t=0}^T \sum_{k=1}^K cf_{kt} \cdot x_{kt}. \quad (2.8)$$

In each period of our simulation the value of each risk factor in x_{mt} is given by a stochastic draw from a known set of historical values

$$\langle x_{mt} \rangle = P(x_m), \quad (2.9)$$

where $P(x_m)$ is the probability distribution of historical values of x_m .

3. Risks Associated with Private Equity Investments

Related research on private equity risk modeling issues, on which our approach could be based, is relatively scarce. Weidig and colleagues (2005) used a Monte-Carlo simulation on PEF in order to investigate the diversification benefits of investing in a FoF instead of a PEF based on the standard deviation of IRRs and Multiples. The result was that the standard deviation of the return of a private equity investment can be significantly reduced by investing in a FoF instead of an ordinary PEF. Malherbe (2003) modeled PEF by approximating commitment drawdowns, the investment values, and return repayments with lognormal, and Squared Bessel processes. This model was then tested for its ability to predict the cash flows of private equity investments. Additionally, Kaplan and Schoar (2005), as well as Ljungqvist and Richardson (2003) conducted research that helped to justify some of the fundamental assumptions regarding PEF that were used throughout the simulation process. Their basic results indicate that results on private equity investments net of fees approximate the returns on the S&P 500, but the actual returns strongly depend on the PEF manager.

Our main argument is that private equity investments and associated risks are somewhat different from most other financial investments. Accordingly, in the following paragraph we discuss the presumably most important types of risks using the investment chain sketched above.

First of all, there is the *cash flow* risk which describes the uncertainty associated with the size and the timing of the actual cash flows. Within the surveyed environment, this risk exists on the level of the PEF and on the level of the FoF. Although, on the level of the PEF, the investment cash flows from the PEF to the PC are stipulated during the negotiations for the investment contract, these cash flows are far from being deterministic. They are controlled by the actual capital burn rate of the portfolio companies and the achievement of certain milestones, which are both hardly influenced by the PEF (no. 3 in Figure 1). The uncertainty associated with the capital distributions from the portfolio company to the PEF is even greater because these cash flows (no. 4 in Figure 1) are heavily dependent on the performance of the PCs and on external factors like the conditions of the financial markets.

On the level of the FoF, the exposure to the cash flow risk is even stronger. The cash flows from the FoF to the PEF are influenced by the unknown capital needs of all underlying portfolio companies of the PEF (no. 2 in Figure 1). Additionally, the capital distributions from the PEF to the FoF are as unpredictable as they are between the PEF and the PCs (no. 5 in Figure 1).

Liquidity risk describes an investor's risk of not being able to exit an investment in face of an impending loss (no. 6 in Figure 1, see [6]). As PEF are usually not publicly listed, there are few possibilities to exit an investment in a FoF fund prior to its liquidation. In fact, transactions on the secondary market on which private equity investments may be sold prior to their liquidation are estimated to require a

discount of up to 70% on the fair value of the investments (see [8]).^e Hence, liquidity risk is of high importance for an investor in a FoF.

Especially for leveraged buyout transactions dealt with in the present paper, *leverage and interest rate risk* are of great importance. An increase in the interest rates, and therefore in the cost of debt, may cause diminishing profits for the portfolio companies. Ceteris paribus, in such a case higher leverage increases the probability of a target company's default. In addition, capital that is called by a fund but not yet invested or capital that has been paid back to the fund but is not yet distributed to the investors is kept in an interest-bearing account (I_{FoF} and I_{PEF} in figure 1). Consequently, the interest paid on this capital depends on the current market conditions. This interest, however, is part of the investors' return and hence influences the fund's performance.

Currency risk describes the uncertainty associated with the changes in exchange rates of two different currency regions. Within private equity investments, currency risk arises if any of the participants of an investment, namely the investor, the FoF, the PEF, or the PC, are not located in the same currency region. Any cash flows that therefore need to be exchanged in a different currency are subject to this risk. It may be hedged on the part of the investor, which, however, is a rarely used measure in private equity investments. The reason is the difficulty of implementing such constructs for private equity investments, because the investor would need very detailed information on the investments of the funds.

Valuation risk derives, among others, from the lack of binding valuation standards for the private equity industry. Hence, reported valuation figures may be different depending on the motivation of the respective report. Another problem is the difficulty of determining the real value of the PCs (see [6]) as they are normally not listed on a stock exchange and therefore, on average, do not have a transparent market price.

Style drift risk describes the possibility that a fund manager deviates from the specifications agreed upon in the investment contract and hence changes the risk-return structure of the investments (see [1]). This may have different consequences for the investor: The new risk may be simply too high or too low for an investor's preferences. Alternatively, the new investment style may not fit into an investor's overall portfolio and, therefore, may cause a deviation from the efficient frontier.

Manager risk, finally, is defined by the strong dependence of PEF on the investment skills of the fund managers. This risk consists of two parts: First, it is necessary for the investor to identify the best fund manager for the purpose conceived. Second, after choosing a fund manager, this manager has to be monitored in order to keep her/his actions in line with the investor's interests.

4. Model of Private Equity Investments

The basic idea of our research is to simulate a sufficient number of hypothetical private equity investments in order to assess the VaR as a measure of risk associated with these investments. For this, we use a Monte-Carlo simulation approach since other approaches, such as historical approaches or variance-covariance approaches

^eWithin the modeled investment structure, secondary transaction may take place in two different ways, depending on the investment level. The first is the sale of a stake in a PC from one PEF to another. The second is the sale of a stake in a PEF from one FoF to another.

seem less appropriate for private equity investments.^f

Our Monte-Carlo simulation first integrates historical data on private equity investments into a stochastic process. Then, scenarios are created for all the risk factors. By randomly creating new values for the scenarios according to the stochastic processes, a generic distribution of results can be generated (see [13] and [4]). The simulation was designed to follow the cash flows in the private equity FoF investment cycle as closely as possible. The general structure of the implemented model is depicted in Figure 2.

To begin with, European and US American portfolio companies in our sample of historical transactions of PEF are separated into two subsamples. Because US American investments are noted in Euro in the original sample, these cash flows are reconverted into US Dollars based on historic exchange rates (1). This step is necessary to account for currency risks in private equity investments. In a second step, these investments are randomly assembled to form new hypothetical PEF. For European and US American PEF, the pool of potential investments consists only of transactions of the respective region. This assumption is based on two premises: First, PEF commonly do show such a regional focus. And second, this structure is able to construct the desired regional orientation of the FoF most accurately. The assembly is executed completely randomly with every potential investment from the respective region in the sample having the same probability of being selected. Additionally, once an investment is included in a fund, an investment lag is generated. This lag simulates the time a PEF needs to find and close promising deals. In reality, this delay may be quite significant for private equity investments as identifying the 'good' deals is a very time consuming task.

The detailed calculation of cash flows (as shown in Figure 2) and operationalization of risk categories acting on these capital streams are illustrated below.

4.1. *Cash Flows*

For every transaction i of a private equity fund, the cash flow in month t is written as $cf^{i,t}$. The basic cash flow of a PEF j in month t is then obtained by aggregating the cash flows of the I transactions or investments (step 2 in figure 2)

$$cf_2^{j,t} = \sum_{i=1}^I cf^{i,t}. \quad (4.1)$$

As outlined earlier, the overall cash flows of PEF are not only made up by the cash flows from the investment activities. Any additional monthly cash flows

^fThere are three different ways to estimate the value-at-risk of an investment: historical simulation, variance-covariance approach, and Monte Carlo simulation. The historical simulation acts on the assumption that historical developments of an investment are also representative for the future. The VaR is therefore directly taken from the historical distributions. Correlations are not incorporated as they are assumed to be implicitly included in the considered data. In order to get a representative result with this approach, a huge amount of data is needed that ideally covers a long period of time. The variance-covariance approach assumes that the influencing market factors are normally distributed. The VaR may hence be calculated using the mathematical properties of this distribution (see [13] and [4]). Both of the necessary requirements for historical simulation are not easily fulfilled for private equity investments, due to the lack of disclosure requirements for the funds and the comparatively short history of private equity in general. The variance-covariance approach would need the returns on private equity investments to be normally distributed. This, however, is surely not the case in reality (Ick 2005, p.14).

are calculated in the next step (3). Secondary investments are sold by a PEF as a secondary transaction whenever cash flows lie beyond the maximum allowable lifetime of a fund. The resulting secondary cash flows of the j -th PEF $cf_{Sec}^{i,t}$ are zero for any month that lies within the lifetime T^j of PEF j . In month T^j , the cash flow is composed of all cash flows of the I PCs in PEF j that lie beyond T^j discounted with the rate r and multiplied with a discount rate for secondary transactions d^{sec} . For every month t after T^j (i.e. all months $t > T^j$), the cash flows from secondary transactions of PEF j equal the inverted gross cash flows, so that all cash flows beyond the lifetime of a fund equal zero

$$cf_{3a}^{j,t} = cf_2^{i,t} + \sum_{i=1}^I cf_{Sec}^{i,t}, \text{ with} \quad (4.2)$$

$$cf_{Sec}^{i,t} = \begin{cases} 0 & \text{if } t < T^j \\ \sum_{u=T^j+1}^{\infty} \frac{\sum_{i=1}^I cf^{i,T^j+1}}{(1+r)^{u-T^j}} \cdot d^{Sec} & \text{if } t = T^j \\ -\sum_{i=1}^I cf^{i,t} & \text{if } t > T^j \end{cases} \quad (4.3)$$

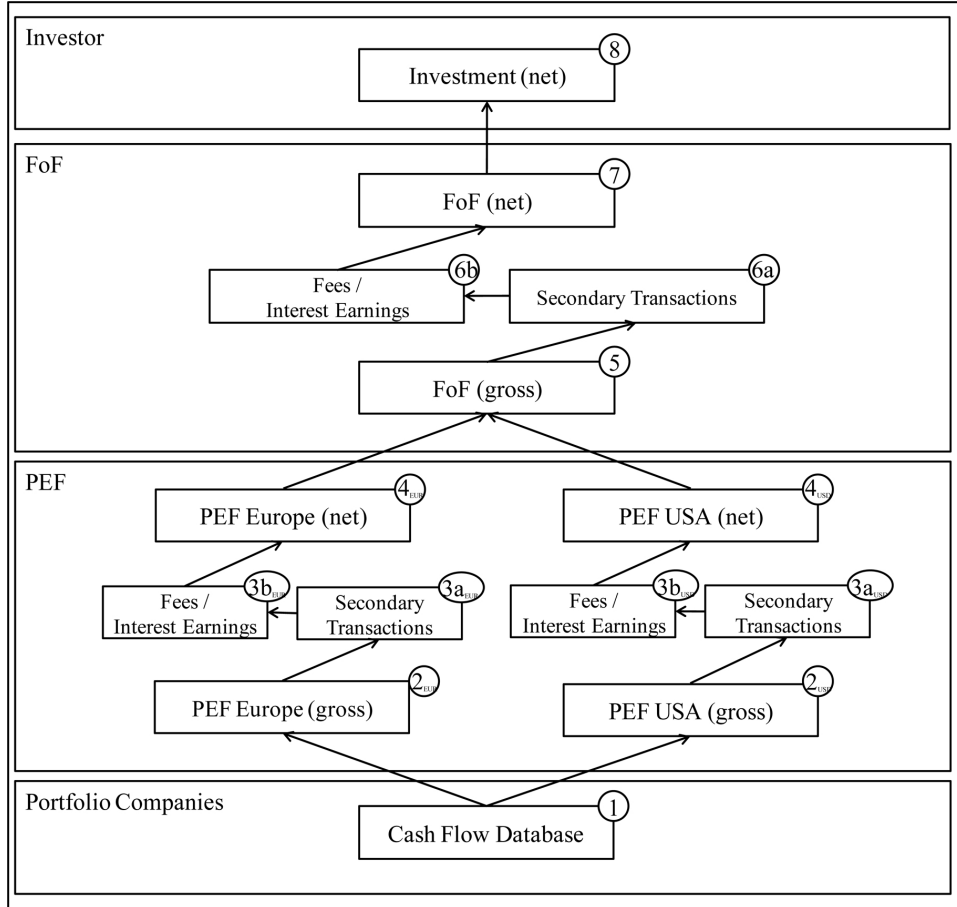


Fig. 2. Model of the private equity investment cycle.

Based on this gross cash flow, composed of investments and secondary transactions, the management fee as remuneration for the PEF managers is deducted. During the investment period IP^j , which is the timeframe scheduled for the investment activity of the fund managers, the full management fee m_{IP}^j accrues based on the committed capital of the PEF j at this point in time $CC^{j,t}$. For the calculation of the management fee m_L^j it is assumed that, starting right after the investment period, the invested capital is linearly decreased to zero at the end of a fund's lifetime. Hence, after the investment period and until the liquidation date of the fund, the management fee m_L^j is calculated based on the remaining invested capital of the PEF in this period $IC^{j,t}$, with $IC^{j,t} = (t - IP^j) \cdot \frac{IC^{j,IP}}{L^j - IP^j}$, where term $\frac{IC^{j,IP}}{L^j - IP^j}$ represents the decrease of invested capital assumed in each t in the period between the end of the investment period IP^j and the liquidation of the fund L^j , and $(t - IP^j)$ simply expresses the number of t passed since the end of the investment period.[§]

The liquidation date L^j of a fund is the lower value of the highest maturity of the portfolio companies, including the delays of the fund's investments and the maximum allowable lifetime of the PEF.

$$cf_{3b'}^{j,t} = cf_{3a}^{j,t} - m^{j,t}, \text{ with} \quad (4.4)$$

$$m^{j,t} = \begin{cases} m_{IP}^j \cdot CC^{j,t} & \text{if } t \leq IP^j \\ m_L^j \cdot IC^{j,t} & \text{if } IP^j < t < L^j. \end{cases} \quad (4.5)$$

Carried interest CI^j is deducted if a fund earns an IRR^j that is higher than the hurdle rate H^j based on $\sum_{t=1}^{\infty} cf_{3b'}^{j,t}$. The carried interest is then calculated based on the respective positive cash flows $cf_{3b'}^{j,t}$ multiplied with the carried interest rate ci^j of private equity fund j

$$cf_{3b''}^{j,t} = cf_{3b'}^{j,t} - c^{j,t}, \text{ for all } t \text{ in which } cf_{3b'}^{j,t} > 0, \text{ with} \quad (4.6)$$

$$c^{j,t} = \begin{cases} 0 & \text{if } IRR^j < H^j \wedge cf_{3b'}^{j,t} < 0 \\ ci^j \cdot cf_{3b'}^{j,t} & \text{if } IRR^j \geq H^j. \end{cases} \quad (4.7)$$

At this point, in order to keep PEF management efficient, monthly cash flows are summed up into tranches. There is a differentiation between tranches for positive cash flows or capital distributions $tr_+^{j,t}$ and for negative cash flows or capital calls $tr_-^{j,t}$.

PEF as closed end funds are not allowed to reinvest capital, i.e. use positive cash flows for further transactions. Hence, PEF managers set a minimum size for tranches of capital distributions TR_D^j . Whenever the accumulated monthly positive net cash flows exceed this threshold, these are distributed to the investors of the

[§]The assumption of linearly decreasing invested capital is surely a simplifying and rather unrealistic assumption. However, we chose this implementation in order to keep the modeling structure manageable. This simplification - compared to the actual procedure of calculating the management fee based on the actually invested capital - was necessary as the invested capital depends, besides the actual cash flows, on legal issues. These regulate which part of the redistributions is treated as capital gain and which part is treated as repayment of invested capital.

PEF. After this has happened, say in month q_1 , the positive cash flows are again gathered until they exceed TR_D^j in month q_2 , and so forth. In all months in-between, the monthly tranche payment is zero. This can be expressed mathematically:

$$tr_+^{j,t} = \begin{cases} 0 & \text{if } \sum_{q_1+1}^{q_2} cf_{3b''}^{j,t} < TR_D^j \wedge cf_{3b''}^{j,t} > 0 \\ \sum_{q_1+1}^{q_2} cf_{3b''}^{j,t} & \text{if } \sum_{q_1+1}^{q_2} cf_{3b''}^{j,t} \geq TR_D^j. \end{cases} \quad (4.8)$$

In turn, a negative tranche $tr_-^{j,t}$ accrues in month q_2 , i.e. capital is called from the investors into a PEF, when the accumulated capital spent exceeds the negative tranche since the last capital call in month q_1 . In contrast, the negative tranche or capital call in any month t equals zero if the accumulated monthly negative cash flows since the last capital call is smaller than the last capital call. However, PEF managers, again for reasons of efficiency, set a minimum size for capital calls TR_{CC}^j . Hence, the actual capital called results from multiplying this amount TR_{CC}^j with a factor $ac^{j,t}$ that represents the next highest integer of the additional capital that is needed $CN^{j,t}$ in month p_1 divided by TR_{CC}^j . This reflects the fact, that it is potentially possible that the fund may need more money in a month than it would receive by calling the minimum size of a capital call.

$$tr_-^{j,t} = \begin{cases} 0 & \text{if } |\sum_{p_1+1}^{p_2} cf_{3b'''}^{j,t}| < TR_{CC}^j \wedge cf_{3b'''}^{j,t} < 0 \\ TR_{CC}^j \cdot ac^{j,t} & \text{if } |\sum_{p_1+1}^{p_2} cf_{3b'''}^{j,t}| \geq TR_{CC}^j \wedge cf_{3b'''}^{j,t} < 0, \text{ with} \end{cases} \quad (4.9)$$

$$ac^{j,t} = \frac{CN^{j,t}}{TR_{CC}^j} \text{ and } CN^{j,t} = \left\lceil \sum_{p_1+1}^{p_2} cf_{3b'''}^{j,t} \right\rceil. \quad (4.10)$$

Finally, in order to obtain the net cash flows $cf_{net}^{j,k,t}$ of PEF j in month t to FoF k as investor, three further steps need to be taken. The first one is simply to add up the positive and the negative tranches. Secondly, these cash flows are multiplied with a factor $s^{j,k}$ that represents the share of the FoF k in this specific PEF j . This share $s^{j,k}$ is determined by dividing the target commitment TC_{reg}^k of each PEF by the sum of the negative tranches of that PEF. The target commitment TC_{reg}^k in turn consists of the total capital available for investments in the respective region (either Europe or the USA) divided by the number of regional PEF J_{reg}^k the FoF has invested in. The total capital available for investments is calculated using the total investment volume of the FoF for that region TIV_{reg}^k multiplied by the investment ratio of the FoF IR^k .

This number is then deducted from the regional share of the management fee of the respective FoF $m_{reg}^{k,t}$. This number reflects the share of the investment volume of that region compared to the other region. Hence, the target commitment results from

$$TC_{reg}^k = \frac{(TIV_{reg}^k \cdot IR^k) - m_{reg}^{k,t}}{J_{reg}^k}. \quad (4.11)$$

The third and final step then is to add the interest earnings $ie^{j,t}$ of PEF j in month t that are gained for capital called but not instantly invested (4). This leads to

$$cf_{net}^{j,k,t} = s^{j,k} \cdot \left[t_+^{j,t} + t_-^{j,t} \right] + ie^{j,t}, \text{ with} \quad (4.12)$$

$$s^{j,k} = \frac{TC_{reg}}{\sum_{t=0}^T tr_-^{j,t}}. \quad (4.13)$$

This net cash flow from PEF to FoF corresponds conceptually to the cash flow $cf_2^{i,t}$ on the investment level, which represents the actual gross cash flow from the PC to the PEF. Here, $cf_{net}^{j,k,t}$ (could equally be denoted as $cf_4^{j,k,t}$, continuing the logic of numbering cash flow streams) represents the original cash flows from FoF investments into PEF. The sum of the net cash flows across all J^k private equity funds constitute the gross cash flow $cf_5^{k,t}$ of the FoF (5)

$$cf_5^{k,t} = \sum_{j=1}^{J^k} cf_4^{j,k,t}. \quad (4.14)$$

At the FoF level the same logic applies for additional cash flow streams: Firstly, FoF generate additional cash flows $cf_{sec}^{j,t}$ from secondary transactions. With T^k denoting the lifetime of FoF k , the same logic as for the calculation of cash flows $cf_{3a}^{j,t}$ from secondary transactions at the PEF level leads to:

$$cf_{6a}^{k,t} = cf_5^{j,t} + \sum_{j=1}^J cf_{Sec}^{j,t}, \text{ with} \quad (4.15)$$

$$cf_{Sec}^{j,t} = \begin{cases} 0 & \text{if } t < T^k \\ \left[\sum_{u=T^k+1}^{\infty} \frac{\left(\sum_{j=1}^J cf^{j,T^k+1} \right)}{(1+r)^{u-T^k}} \right] \cdot d_{-Sec} & \text{if } t = T^k \\ - \left(\sum_{j=1}^J cf^{j,t} \right) & \text{if } t > T^k. \end{cases} \quad (4.16)$$

The integration of the management fee, the carried interest, the tranches, and the interest earnings on the level of the FoF is also is analogous to the PEF. The only main difference is that the specifications of the PEF have to be exchanged for those of the FoF (6).

The management fee $m^{k,t}$ of FoF k is calculated as (with $CC^{k,t}$ being capital committed, $IC^{k,t}$ being the remaining invested capital calculated by $IC^{k,t} = (t - IP^k) \cdot \frac{IC^{k,IP}}{L^k - IP^k}$, L^k being liquidation date of the FoF, and IP^k being the investment period)

$$cf_{6b'}^{k,t} = cf_{6a}^{k,t} - m^{k,t}, \text{ with} \quad (4.17)$$

$$m^{k,t} = \begin{cases} m_{IP}^k \cdot CC^{k,t} & \text{if } t \leq IP^k \\ m_L^k \cdot IC^{k,t} & \text{if } IP^k < t < L^k. \end{cases} \quad (4.18)$$

Accordingly, the cash flows after carry for each FoF k in month t can be easily derived from the calculation of $cf_{3b'}^{j,t}$ at the PEF level (being IRR^k the internal rate of return, H^k the hurdle rate, and ci^k the carried interest)

$$cf_{6b''}^{k,t} = cf_{6b'}^{k,t} - c^{k,t}, \text{ for all } t \text{ in which } cf_{6b'}^{k,t} > 0, \text{ with} \quad (4.19)$$

$$c^{k,t} = \begin{cases} 0 & \text{if } IRR^k < H^k \\ ci^k \cdot cf_{6b'}^{k,t} & \text{if } IRR^k \geq H^k \wedge cf_{6b'}^{k,t} > 0. \end{cases} \quad (4.20)$$

Similarly, the tranches of capital distributions $tr_+^{k,t}$ and capital calls $tr_-^{k,t}$ at the FoF level are analogously computed to the PEF level (TR_d^k being the minimum size for tranches of capital distributions, TR_{CC}^k being the minimum size for capital calls, and $ac^{k,t}$ being the next highest integer of additional capital needed $CN^{k,t}$)

$$tr_+^{k,t} = \begin{cases} 0 & \text{if } \sum_{q_1+1}^{q_2} cf_{6b''}^{k,t} < TR_D^k \wedge cf_{6b''}^{k,t} > 0 \\ \sum_{q_1+1}^{q_2} cf_{6b''}^{k,t} & \text{if } \sum_{q_1+1}^{q_2} cf_{6b''}^{k,t} \geq TR_D^k \wedge cf_{6b''}^{k,t} > 0. \end{cases} \quad (4.21)$$

$$tr_-^{k,t} = \begin{cases} 0 & \text{if } |\sum_{p_1+1}^{p''} cf_{6b''}^{k,t}| < TR_{CC}^k \wedge cf_{6b''}^{k,t} < 0 \\ T_{CC}^k \cdot ac^{k,t} & \text{if } |\sum_{p_1+1}^{p''} cf_{6b''}^{k,t}| \geq TR_{CC}^k \wedge cf_{6b''}^{k,t} < 0, \text{ with} \end{cases} \quad (4.22)$$

$$ac^{k,t} = \frac{CN^{k,t}}{TR_{CC}^k} \text{ and } CN^{k,t} = \left\lceil \sum_{p_1+1}^{p_n} cf_{6b''}^{k,t} \right\rceil. \quad (4.23)$$

The only remaining step in order to get to the monthly net cash flows from FoF k to investor l in month t (8) is to account for the share of the investor in the FoF and add the interest earnings for capital called but not instantly invested

$$cf_{net}^{k,l,t} = s^{k,l} \cdot \left[t_+^{k,t} + t_-^{k,t} \right] + ie^{k,t}. \quad (4.24)$$

4.2. Risk Categories

The risks that are considered in the present model are cash flow risk, currency risk, interest rate risk, and leverage risk. *Cash flow risk* is considered by randomly creating new funds. Real private equity transactions are drawn from the sample to build hypothetical new FoF and private equity funds. By this random procedure, the sample characteristics are transferred to the modeled funds.^h Besides the pure drawing of transactions, the second random element that contributes to creating new funds is accounting for investment delays on both fund layers. At the level of the private equity funds, a delay is simulated for each individual transaction as it describes the time a fund needs to identify and close the respective deal. In order to get this delay as realistically as possible, the corresponding distribution was derived from a study conducted by Ljungqvist and Richardson (2003). Among other aspects, they investigated the capital calls of PEF on a yearly basis. As in this case, the density function of the investment delays are needed on a monthly

^hIn this context, "randomly" means that every investment is drawn with exactly the same probability. This probability was hence set to with $p_i = \frac{1}{N_{reg}}$ being the probability that transaction i is chosen and N_{reg} representing the total number of transaction for the respective region.

basis, these yearly figures are approximated by the following polynomial regression of the sixth degree:ⁱ

$$P_t = 3.0 \cdot 10^{-13} \cdot t^6 - 1.22 \cdot 10^{-10} \cdot t^5 + 1.8 \cdot 10^{-8} \cdot t^4 - 1.2 \cdot 10^{-6} \cdot t^3 + 2.3 \cdot 10^{-5} \cdot t^2 + 8.0 \cdot 10^{-5} \cdot t + 1.4 \cdot 10^{-2} \quad (4.25)$$

with P_t being the probability of a delay of t months.

To the authors' knowledge, a study covering the delay on the level of the FoF does not yet exist. The assumption that FoF, exhibit the same investment delays as private equity funds does not produce realistic simulation results, as this would lead to extremely high total delays (aggregated over both layers) for some of the PCs.^j The function determining the investment delay on the level of the FoF was hence limited to two years in a way that the original pattern of Ljungqvist and Richardson (2003) was compressed to fit this confinement.

Theoretically, a private equity investment cycle might be subject to a wide array of *currency risks*. However, since the present model only contains US American and European investments, the only relevant currency risk exists between US dollar and Euro.^k This limitation was imposed to reduce the complexity of the model and is justified by the fact that these two regions cover the vast majority of all private equity activities during recent years; in fact, 83.9% of all global private equity investments from 1998 to 2005 were done in these two areas (see [20]). However, the model only integrates currency risk on the level of FoF. For reasons of simplification, it is assumed that a US PEF only invests within the US currency area whereas a European PEF solely focuses on European PCs.^l Accordingly, the euro cash flows are exchanged into US dollars and vice versa, whenever a cash flow is transferred between a FoF in one area and a private equity fund in the other.

For every simulated period, the model generates a new market condition for the exchange rate based on historical data which is then applied to the respective cash flows. For example, an investment whose historical return was decreased by the development of the exchange rate may now be faced with the opposite environment. This adds further realism to the results.

The current exchange rate is determined based on a predefined initial exchange rate. The change in the starting month's exchange rate and that of the next month is simulated using a normal distribution that mimics the historical distribution of changes in the exchange rate. This historic sample of changes in exchange rates contains monthly data ranging from January 1970 to February 2008 as reported by the Deutsche Bundesbank.^m The change in exchange rates is calculated using the formula:

ⁱThe comparison with the original data leads to a R^2 -coefficient of determination of 98.5%.

^jThe total delay of a portfolio company consists of the investment delay of the private equity fund and the investment delay of the fund-of-funds.

^kEven though Great Britain is the most important region within Europe regarding the private equity industry, the British Pound was not included into the simulation as an additional currency region. This is mainly due to the low number of PCs within the data sample that were located in Great Britain. A Monte-Carlo simulation would hence not have been feasible.

^lThis assumption might have to be revisited going forward due to the increasing internationalization of PEF.

^mBefore the introduction of the Euro, the exchange rate between US-Dollar and Deutsche Mark was used instead. This focus on the German market was used as the majority of the European Data Sample of portfolio companies is located within Germany. Alternatively, the change in exchange

$$\Delta_t = \frac{E_{t+1} - E_t}{E_t}, \text{ with} \quad (4.26)$$

Δ_t being the change in exchange rates and E_t being the exchange rate in month t .

The approximated normal distribution has a mean of 0.003 and a standard deviation of 0.026. The Shapiro-Wilk test resulted in a p-value of 0.022 at a 99% confidence level. This indicates that the historic sample most likely originates from a normally distributed population. Consequently, the simulation uses the normal distribution to generate new exchange rate developments. The mean of this distribution, however, is set to zero as there is no justification that one currency should perform better than the other on average. The difference is hence treated as a statistical inaccuracy.

The approach chosen to model the *interest rate risk* affecting investor money kept in an interest-bearing account is quite similar to the simulation of the exchange rate. Starting with an initial level, the change in the interest rate is simulated using the historical data of the European Interbank Offered Rate (EURIBOR) and its antecedent products.ⁿ The changes were approximated by a normal distribution with a mean value of 0.0021 and a standard distribution of 0.0465. The Shapiro-Wilk test at a 99% confidence level of these two distribution results in a p-value of 0.4, which indicates that, most likely, both distributions were taken from the same population.

Again, the mean value of the normal distribution is zero for the simulation, as, on average, interest rates should not exhibit a drift in any direction. First trial runs of the simulation indicated an unrealistic tendency to create interest rates that exceeded common levels significantly. Hence, a soft barrier was implemented to prevent these developments and maintain the realism of the model: Whenever the interest rate exceeds the value of ten percent, the distribution for the change in that interest rate was shifted to a mean value of -1.7%, which equals the historical mean for interest rates above this level.

The simulation of the *leverage risk* is inherent in the data sample that is used for the simulation. The data includes investments that were structured using different proportions of debt. Consequently, the historical default rates already account for leverage risk. Following the trade-off theory, deals that were structured using high proportions of debt are exposed to higher financial risk and should, on average, show a higher probability of default and vice versa.

The suggested model does not incorporate *liquidity risk*, *valuation risk*, *style-drift risk*, and *manager risk*. Even though liquidity risk is of great importance for an investor in a private equity FoF, the liquidity risk of a specific investment depends strongly on the remainder of the investor's portfolio and its ability to create additional cash. Hence, liquidity risk needs to be estimated based on the total portfolio. *Valuation risk* is of no relevance for this analysis as the valuation is performed ex-post, after all investments are realized and, hence, there is no associated uncertainty

rates between the ECU, which consisted of a weighted basket of the currencies of the European Union, and US-Dollar could be analyzed to get a broader view on the whole European area.

ⁿFrom January 1989 to June 1990, this is the Geldmarktsatz am Frankfurter Bankplatz. From July 1990 to December 1998, it is the London Interbank Offered Rate. The EURIBOR starts from January 1999.

with future cash flows.

Style-drift risk is not simulated because portfolio companies are drawn randomly from the available pool of historical transactions. Accordingly, the simulated FoF have an investment style that, on average, reflects the composition of the sample. Hence, within the given framework, there is no style-drift risk. Simulating FoF that only invest in funds of a certain investment style would require a much larger sample in order to properly run a Monte-Carlo simulation. The rationale for omitting the *manager risk* from the model is quite similar to that of the style drift risk. As the simulation tool randomly creates new FoF and PEF, some of these funds will, ex post, turn out to be managed by proper managers and some of these funds will not. The share of 'good' funds will therefore, as investments are drawn randomly, depend on the share of 'good' investments within the sample. The question of how close to reality the simulated funds are, therefore depends on how well the used data maps the actual market.

A short overview of the modeling approach to the different risk classes that were integrated is depicted in Figure 3.

Potential correlations among and within the different risk categories were also investigated. Specifically, this concerns three kinds of correlations: the intertemporal autocorrelation of exchange rates and interest rates as well as the correlation between exchange rate and interest rate. The autocorrelation of exchange rates has been subject to excessive research (e.g. [18] and [16]). Nonetheless, forecasts of random-walk models are still not outperformed by any of the more sophisticated models (see [12]). Accordingly, exchange rates are simulated assuming random-walk properties. These properties are twofold: All elements need to be part of the same population and each element needs to be independent from one another. The values considered in this context are the changes in the exchange rate between two months. As these values are required to be independent from one another, autocorrelation is set to zero. The autocorrelation of interest rates was calculated using the standard formula of autocorrelation:

Risk Category	Cash Flows*							Modeling within Simulation Tool
								
Cash Flow Risk	X							Monte-Carlo simulation of PE FoFs is used to estimate timing and size.
	X							
Currency Risk				X				Exchange of cash flows on basis of simulated exchange rate.
Interest Rate Risk			X					Capital that is kept in an interest-bearing account earns the market interest rate.
			X			X		
Leverage Risk							X	Monte-Carlo simulation of PE FoFs recognizes historical default rates.

** Numbers correspond to those in Figure 2.*

Fig. 3. Overview of the integration of the risk categories into the simulation model.

$$p_{x,y} = \frac{Cov(X,Y)}{\sigma_x \sigma_y} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \mu_x)(y_i - \mu_y)}{\sigma_x \sigma_y}, \quad (4.27)$$

with i being the number of attributes.

Subscript 'x' is the original time series of changes in the interest rates. Subscript 'y' refers to the same time series delayed by one month. Accordingly, the correlation in the changes of the interest rate of two consecutive months in the sample is 47.1% and was used for the simulation process.

The correlation between interest rates and exchange rates is theoretically explained using interest rate parity.^o This model claims that interest rates and exchange rates should exhibit a correlated movement. However, empirical evidence has not sufficiently proved this behavior (see [7]). In fact, some results even suggest contradictory behavior. Accordingly, integration of correlations between interest and exchange rates seemed not advisable.

5. Simulation Data and Parameters

5.1. Sample and Further Input Variables

Given the fact that in simulations the characteristics of the original sample have a strong impact on the results, a detailed description of the initial data set is indispensable. The sample used for the simulation consists of longitudinal monthly cash flows of leveraged buyouts in the U.S. and Europe. It was collected by an FoF manager during the due diligence process of potential target funds. The data ranges from April 1977 to February 2008.

The data provided contains the cash flows of the investments of PEF on a monthly basis and gross of any fees. In addition to these cash flows, the sample contains further important information about each investment. Complete information on all investments is available for the investment date, the region of the investment (U.S., Europe, and Asia), and the realization status. Further comprehensive information could be gathered using the cash flows of the investments. This refers to the investment amount, realized and unrealized proceeds, the IRR and the multiple of the investment.

The initial sample contained 4032 portfolio companies. 26 investments were excluded due to a regional orientation outside of Europe or the U.S. and 1319 investments were not yet realized at the time of the respective reporting. As these investments were historically usually valued at cost, they do not reflect the ex-post observable cash flow pattern of private equity investments. These exclusions result in a final sample used for the simulation of 2687 investments.

Table 1 displays additional descriptive variables of the data that was used for the simulation runs.

^oFor further documentation on this topic, see [7], p. 1.

Table 1. Descriptive statistics of the data sample that was used for the simulation.

Year of Investment	No. of Transactions		Amount Invested		DIC (capital weighted)		IRR (capital weighted)		Shortfall Probability ^a	
	EU	US	EU	US	EU	US	EU	US	EU	US
1977	0	3	- €	4.403.145,42 €	n/a	21,17	n/a	45,5%	n/a	0,0%
1978	0	0	- €	- €	n/a	n/a	n/a	n/a	n/a	n/a
1979	0	15	- €	42.254.615,46 €	n/a	3,29	n/a	33,1%	n/a	46,7%
1980	0	10	- €	17.869.722,00 €	n/a	3,29	n/a	48,9%	n/a	50,0%
1981	0	30	- €	386.889.589,00 €	n/a	3,88	n/a	29,1%	n/a	33,3%
1982	0	11	- €	13.051.121,00 €	n/a	1,73	n/a	26,8%	n/a	54,5%
1983	8	24	31.002.000,00 €	197.934.831,00 €	-0,10	2,53	-3,8%	46,1%	25,0%	50,0%
1984	10	44	37.683.150,00 €	812.744.476,09 €	1,38	4,99	42,4%	42,6%	30,0%	31,8%
1985	16	37	54.678.317,00 €	735.255.408,30 €	1,49	3,36	38,5%	36,7%	37,5%	27,0%
1986	10	61	68.503.645,00 €	872.265.073,37 €	5,55	12,09	24,3%	48,9%	30,0%	26,2%
1987	9	38	40.513.263,00 €	813.845.273,89 €	3,24	1,84	29,6%	14,6%	11,1%	31,6%
1988	11	61	46.664.772,00 €	1.482.245.903,63 €	0,65	6,06	18,9%	34,4%	36,4%	31,1%
1989	11	50	137.083.810,51 €	3.986.738.380,17 €	1,29	0,13	10,3%	1,6%	36,4%	36,0%
1990	33	51	356.444.393,29 €	585.666.684,11 €	1,14	2,38	22,4%	34,8%	21,2%	27,5%
1991	36	50	296.075.723,37 €	1.064.452.565,92 €	1,93	4,04	52,2%	31,1%	27,8%	14,0%
1992	53	62	397.719.776,50 €	1.645.463.692,21 €	1,83	2,81	43,8%	46,7%	15,1%	19,4%
1993	55	78	451.359.550,48 €	1.974.555.738,78 €	1,94	2,19	51,0%	45,8%	14,5%	11,5%
1994	106	66	1.024.282.700,56 €	919.395.390,42 €	1,46	2,07	33,2%	52,5%	21,7%	28,8%
1995	65	87	678.209.031,87 €	2.852.753.907,29 €	2,00	2,00	45,9%	41,0%	20,0%	35,6%
1996	102	83	1.517.805.719,03 €	3.571.638.799,41 €	3,23	1,63	80,3%	43,4%	13,7%	25,3%
1997	110	87	1.633.020.914,25 €	4.134.839.035,03 €	1,45	2,97	32,5%	54,9%	32,7%	25,3%
1998	104	81	2.536.568.604,07 €	4.271.863.454,19 €	1,75	1,21	40,2%	20,0%	31,7%	42,0%
1999	121	122	3.963.020.262,45 €	6.429.890.498,70 €	1,11	0,48	24,8%	10,0%	38,0%	50,8%
2000	144	175	3.913.639.365,17 €	7.872.498.827,53 €	0,54	0,42	12,1%	10,2%	58,3%	62,9%
2001	50	40	2.837.797.133,97 €	1.784.679.302,56 €	1,27	1,34	31,4%	30,3%	22,0%	27,5%
2002	49	42	2.601.979.805,77 €	4.127.918.020,03 €	2,57	1,56	57,6%	46,0%	12,2%	9,5%
2003	39	42	2.358.747.388,24 €	1.734.400.602,71 €	2,50	2,51	81,4%	77,1%	10,3%	16,7%
2004	33	24	1.797.691.673,71 €	2.935.153.966,63 €	3,26	2,47	112,0%	108,4%	12,1%	12,5%
2005	24	6	1.223.703.516,63 €	151.686.919,81 €	2,91	0,73	145,4%	44,6%	29,2%	33,3%
2006	3	2	181.882.970,00 €	99.437.800,00 €	1,06	0,08	46,9%	17,0%	66,7%	0,0%
2007	3	0	34.571.688,00 €	- €	0,11	n/a	15,1%	n/a	66,7%	n/a
1205	1482		28.220.649.174,87 €	55.521.792.744,66 €	1,78	1,85	34,4%	32,5%	28,3%	33,5%

^a A shortfall is defined as a portfolio company with a DIC below zero.

An important issue that needs to be addressed is the extent to which the data sample that was used for the simulation is representative of the whole private equity industry. The IRR of the total data sample equals 32.7%. Even though a direct comparison with similar studies is not possible because they only report net IRRs, this number appears to be significantly above the average gross IRR of the industry. Our sample - like many private equity data samples - is likely to suffer from a survivorship bias. The transactions contained in the sample are extracted from due diligence documents of the respective PEF that are in the fund raising stage. This leads to the bias that only funds that expect to raise a next fund and show a comparable track record are selected by the FoF to get into the due diligence process. This successful historic track record of the PEF also translates in an above-average performance of portfolio companies. Additionally, even though transactions from all over Europe are included in the data sample, German transactions are over-represented.

Even though these shortcomings of the data sample may prevent the empirical results from being representative of the whole private equity industry, the conceptual drawbacks should be acceptable. In any case, the basic purpose of the presented model is to generate realistic results based on the used data. Hence, the results obtained in our research mainly serve to validate the correct functioning of the model and to give some further evidence on private equity risk characteristics based on our imperfect sample.

5.2. *Input and Output Variables*

Besides the already specified data base of historical transactions, some further input data is necessary in order to properly simulate private equity FoF. A set of input parameters was chosen that we considered to be the most realistic. This base case, detailed in the following, also serves as starting point for the subsequent sensitivity analyses of key parameters of private equity FoF.

The initial exchange rate was set at one euro equal to 1.27 US dollars; the initial interest rate was set at 4.32%. Both rates reflect the current market conditions at the time the simulation was run. The interest rate was defined as the European Interbank Offered Rate (EURIBOR). The discount rate for private equity investments was set at 10.5% which equals the average annualized return of a ten year investment in the S&P500 index throughout the time horizon of the data sample. Secondary deals were not discounted compared to the actual present value of future cash flows. As the secondary market is a highly inefficient market, it is difficult to assess the actual market prices and hence this value had to be chosen arbitrarily. However, as will be seen later, this value is of no great importance anyway.

The input parameters used for the simulation of the FoF were derived from the products of the FoF manager who also provided the data sample. Therefore, the assumed management fee was 1%; the corresponding investment period was set at five years. The carried interest amounts to 10% with a hurdle rate of 8%. The maximum lifetime of a fund-of-funds is 15 years;^P the investment ratio is 93.6%. Tranches for capital calls and for capital distributions have a minimum size of 3% of the committed capital.^Q

^PThese values are based on interviews with practitioners.

^QValues are based on our assumptions used for the PEF.

On the level of the PEF, the values were chosen as follows: First, the share of European and US funds within the fund-of-funds was set at 50% each.^r A FoF was assumed to be investing in 20 PEF of which ten in Europe and ten in the US. The number of investments per PEF was set at 18 for both regions. The management fee was assumed to be at 1.9% for both regions, with a carried interest of 20% and a hurdle rate of 8%. The investment period spans five years with a maximum term of 12 years for PEF.^s The investment ratio was, in accordance with Ljungqvist and Richardson (2003), presumed at 93.6%. The size of the tranches was chosen at 3%.^t The number of trial runs for this base case scenario was set at 5000.

Given their prevalent importance for every simulation run, the IRR and the mDIC were calculated. As the output parameters are saved throughout the whole simulation, it is possible to derive the respective frequency distribution and therefore also the iRaR and MaR from this data.

5.3. Results and Discussion

The *base case scenario* produced the density functions of the IRR and the multiple depicted in Figures 4 and 5.

The corresponding iRaR at a 95% confidence level amounts to 15.32% while the calculated MaR at the same confidence level is 0.54x. The density distribution of the multiple across the 5000 simulation runs is described by a standard deviation of 0.21, a skewness of 1.10 and a kurtosis of 2.63. The respective values for the IRR amount to 0.05, 0.95, and 3.14. The modified VaR measures for both, IRR and multiple, appear to be relatively high for a private equity FoF investment

^rThis roughly corresponds with the actual market share of European and US-American private equity investments, if only these two regions are included in the calculations.

^sFigures are derived from the data sample.

^tThis number is based on interviews with practitioners.

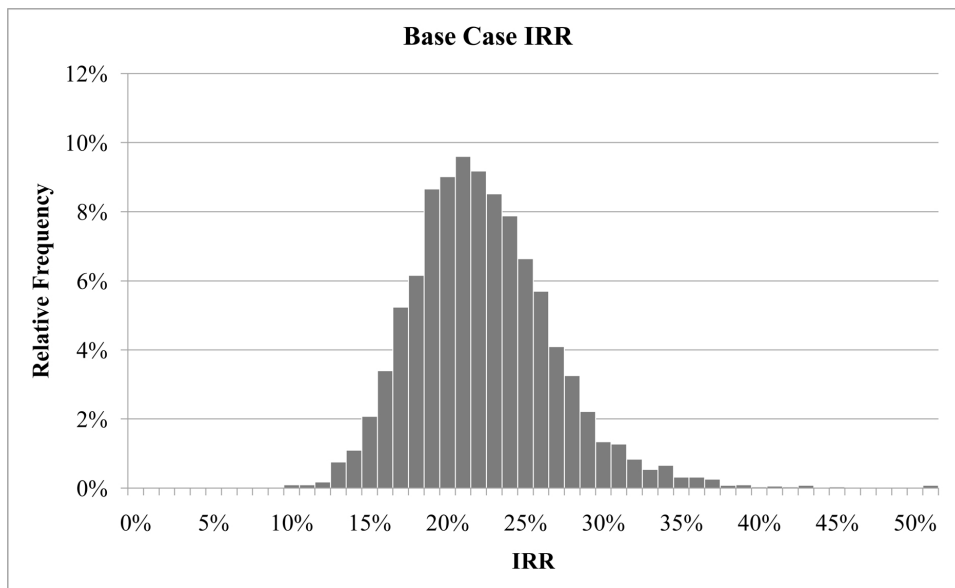


Fig. 4. Frequency distribution of the investors' IRR.

even though the FoF structure adds additional diversification. According to the definition of value-at-risk given above, these results can be interpreted as follows: the probability that an investment into a private equity fund-of-fund yields an IRR less than 15.32% or an mDIC of less than 0.54 is 5%.

There are two major issues about these values: Firstly, there is a lack of representativeness of these results given the caveats against the underlying data sample. Secondly, the modeling approach allows the FoF to choose their investment throughout the entire time period of the data sample, ranging from 1977 to 2007. With a FoF investing in 360 portfolio companies (in our base case a FoF invests in 18 private equity funds which, in turn, invest in 20 portfolio companies) and the relatively high average performance within the data sample, the probability of drawing a really badly performing FoF is low. In reality, however, the private equity industry tends to perform in market cycles with high and low returns, which leads to good and bad investment and vintage years. On the other hand, due to the time-consuming activity of identifying target investments at the level of FoF and the PEF, an FoF indirectly invests in portfolio companies over several years. Assuming that the private equity industry market cycles are shorter than this investment period - as demonstrated in the data set -, the overestimation of iRaR and MaR might not be so severe.

In order to get a feel for the actual impact of these market cycles, we simulated the risk profile with implied market timing. We assumed that it should take an FoF about two years to invest all its capital in private equity funds that in turn commonly have an investment period of about five years. In total, this adds up to a maximum time frame of seven years available for investments in portfolio companies. A scenario was then created including portfolio companies of the seven year period with the lowest IRR in the sample. The importance of market timing was analyzed by comparing the base case against this 'bad timing' scenario. The iRaR for the 'bad timing' scenario is 13.24%, which is 2.08 percentage points lower than the base

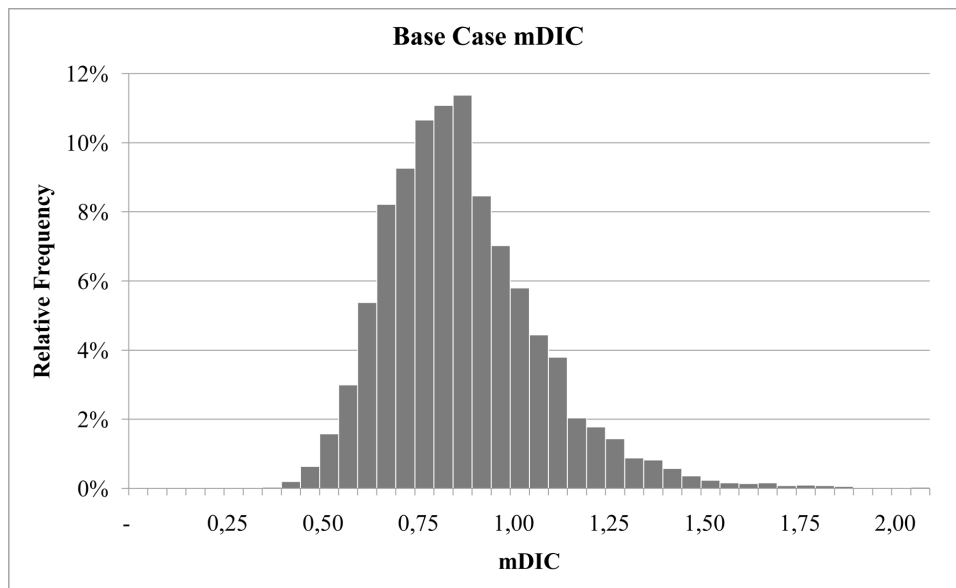


Fig. 5. Frequency distribution of the investors' mDIC.

case iRaR (or -13.57% of the iRaR in the base case).

In addition to the base case scenario, various *sensitivity analyses* were performed. They allow investors to identify the (relative) impact of the change in discretionary, non-random variables (e.g. M_{FoF}) on the performance and risk of a private equity FoF investment. For each driver, sensitivities were calculated for a change in input parameters of 10% upwards and downwards from the base case (default) value. In order to isolate these effects from pure variations through different Monte-Carlo simulations, i.e. different draws, the draws for the base case simulation were stored and reused as the underlying sample for the sensitivity runs. The results of this analysis are illustrated in Figures 6 to 9.

Each of the four figures shows the sensitivities of the iRaR and the mean of IRR as well as the MaR and the mean of multiple induced by central input parameters

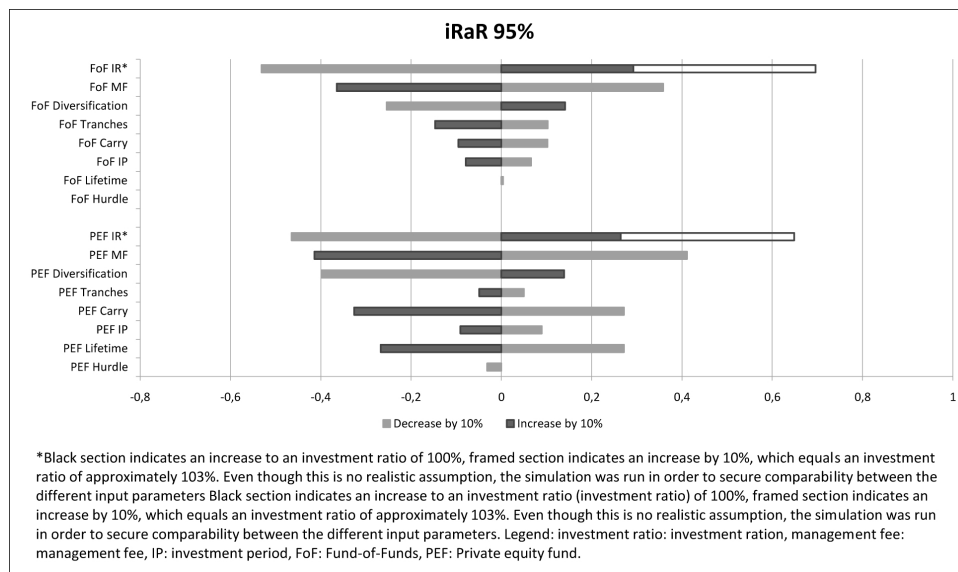


Fig. 6. Impact of changes of input parameters on the iRaR 95%.

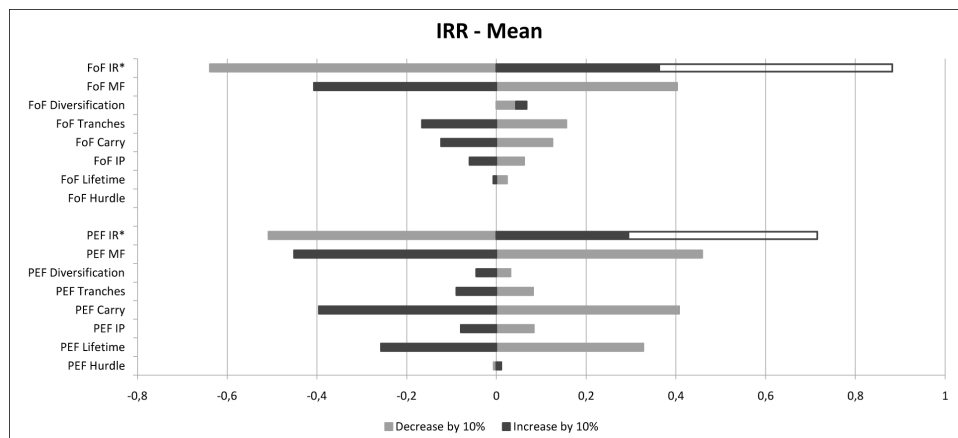


Fig. 7. Impact of changes of input parameters on the MaR 95%.

on the FoF (top) and PEF (bottom) level respectively. The input parameters are ordered with descending impact on the downturn risk of the FoF iRaR (see Figure 6 top). This order also applied to the analysis of the PEF in order to highlight differences between the two investment levels. The black bars indicate the percent alteration in the risk measure triggered by a minus 10% change of the input parameter. In turn, grey bars signal the result of a 10% increase in the relevant driver. In the following, the impact of the changes in the various variables is presented and explained.

There are two general aspects of the results: First, several input parameters cause different changes in the iRaR and MaR depending on whether the parameter was increased or decreased. This leads to the proposition that several of the relationships between the input and the output variable are not linear. Determining the exact shape, however, requires further research. The second observation is that the impact of changes is generally larger for the IRR than for the multiple. This difference is caused by the fact that the multiple only accounts for the size of the CFs whereas the IRR also includes the timing aspect.

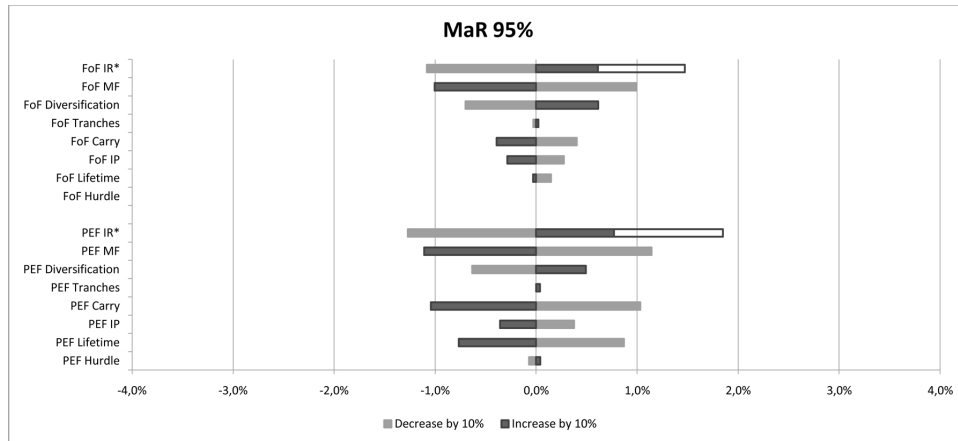


Fig. 8. Impact of changes of the input parameters on the mean of the IRR.

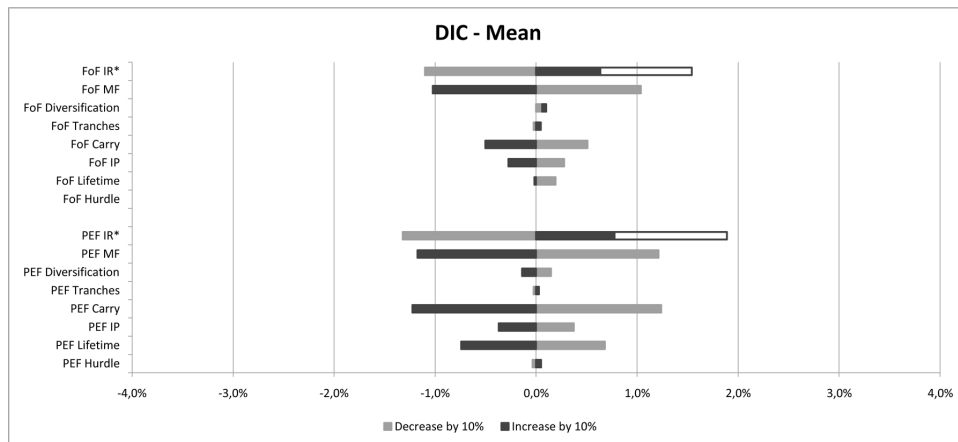


Fig. 9. Impact of changes of the input parameters on the mean of the DIC.

At both investment levels, i.e. the level of the FoF and the PEF, the investment ratio has the strongest impact on the performance measures. For instance, a 10% decrease in the investment ratio on the FoF or the PEF level increases the iRaR by 0.53 or 0.46 percentage points, respectively. For the MaR, the impact for the same change amounts to only 1.1 or 1.3%, respectively. The negative influence of a decreasing investment ratio on the risk of a FoF is due to the management fee which is calculated using the committed capital of the fund and hence a figure that is independent of the actual investment activity of the fund. At least within the investment period, the management fee is a fixed amount. Accordingly, a lower investment ratio increases the relative size of the negative cash flows caused by the management fee. The investment ratio thus influences the return of private equity FoF just as an alteration of the management fee would do.

Consequently, the management fee is quite similar to the investment ratio in its impact on the risk of FoF investments. At both investment levels a change of the management fee has a significant impact on the risk. At the level of the PEF, a change of 10%, i.e. from 2% to 1.8% and 2.2% respectively, results in a shift of about 0.41 percentage points of the iRaR and a 1.1% change for the MaR. At the FoF level, the same change alters the iRaR by about 0.36 percentage points and the MaR by 1.0%. These numbers reflect at least two interesting findings: First, changing the management fee in either direction causes a change in risk of about the same size. This means that, at least in the intervals close to our default values in the base case, risk seems to be linearly dependent on the rate of the management fee. The second conclusion is that, even though FoF have a management fee of about 1% whereas PEF have one of about 2%, the impact is not significantly different. This might be explained by the longer maximum lifetime of FoF compared to PEF. Regarding the management fee, a longer lifetime automatically implies more management fee payments and hence an increased impact.

After investment ratio and management fee, the variable with the greatest impact on the iRaR and MaR is diversification. The impact of diversification on the risk was assessed by altering the number of PEF in an FoF and by altering the number of portfolio companies in a PEF. The impact of these alterations is notable on both investment levels and for both risk measures. The most striking effect can be observed at the PEF level. Here, a decrease of diversification by 10% increases the iRaR by 0.40 percentage points. The influence, however, is not symmetric for positive and negative changes of the diversification. The smaller effect on positive diversification gains is probably due to decreasing marginal diversification effects. These marginal effects probably also explain why the influence at the PEF level is higher than at the FoF level. At the FoF level, the initial value was set at 20 PEF per FoF, whereas at the PEF level, the initial value was set at 18 PCs per PEF. Intuitively, the mean IRR and multiple do not exhibit any notable effects from changing the diversification of a FoF. The arbitrary changes that occurred for the mean are, in the authors' opinion, due to an alteration in the composition of the FoF, as in this case it was not possible to stick to the procedure of using the same draws for every sensitivity analysis, as more or fewer PCs or PEF had to be simulated.

Regarding the size of the tranches of the FoF and PEF, it makes a huge difference whether the iRaR or the MaR are used to assess risk. Changing the size of the tranches for capital calls or capital redistributions has two effects on the

cash flows of the FoF: Larger tranches lead to earlier capital calls and later capital redistributions. Hence they always shift cash flows in a way that is unfavorable for the investor of an FoF. On the other hand, larger tranches also lead to more capital allocated in interest-bearing accounts and, therefore, to higher interest earnings. As the multiple ignores the timing of cash flows, it is not surprising that the MaR hardly reacts to changing tranches. The small impact that remains is caused by the slightly higher interest earnings mentioned before. Accordingly, for the MaR, larger tranches imply less risk. However, the iRaR not only accounts for these additional interest earnings, but also reflects the timing of cash flows. As the return on a bank account is usually lower than the return earned by private equity investments, these additional cash flows are not able to compensate for the less favorable timing. In consequence, contrary to the MaR, larger tranches lead, *ceteris paribus*, to a higher iRaR. The quantitative impact of a 10% change in the size of the tranches is about 0.05 percentage points on the level of the PEF and between 0.1 and 0.15 percentage points on the level of the FoF.

The impact of the carried interest on iRaR and MaR seems to be linear within the tested area. The carried interest on the PEF level has a notably higher influence compared to the FoF level, as the carry rate on the different levels was set at 20% and 10% respectively. The impact of a change of 10% in the carried interest rate amounts to a change of slightly more than 0.1 percentage points for FoF level and between 0.27 and 0.33 percentage points on the PEF level.

The investment period unfolds its impact via the higher management fee that is paid by the FoF and the investors. A longer investment period corresponds to higher management fee payments and lower risk, i.e. increasing the IP by two years leads to two additional years in which the full management fee is due. In consequence, the change of the IP has a similar impact as the investment ratio but at a lower level because payments of management fee are not stopped after the IP, but linearly decreased to zero at the end of the lifetime of the fund. The actual numbers amount to 0.07 to 0.09 percentage points on the iRaR and 0.3 to 0.6% for the MaR.

In our analysis, the hurdle rate exhibits a negligible impact on both levels. This can be explained by the characteristics of the sample. Hence, based on this sample, hurdle rates on both fund levels seem too low to give an incentive to the management on both fund levels.

While the impact of a fund's lifetime (term) is negligible at the FoF level, it is significantly higher at the PEF level. For example, an alteration of a private equity fund's lifetime by 10% (1.2 years) shifts risk by approximately 0.27 percentage points as measured by iRaR. The difference between the two investment levels is due to the higher management fee and shorter default lifetime of the PEF. On both levels, the IP is set to five years in which the management fee is paid in full. From year six to the end of the lifetime of the fund, the management fee linearly decreases to zero. Given the shorter remaining lifetime of the fund and the higher management fee during the IP at PEF level, the marginal decrease in the management fee (per year) is higher than at the FoF level.

In addition to the aforementioned sensitivities we conducted a scenario analyses, exploring the effects of the omnipresent discussion around management fee and carried interest on the private equity fund level. The model shows that a simultaneous reduction in management fee from 2% to 1% and the carried interest from 20% to 10% increases the mean IRR by 4.3 percentage points and the mean mDIC

by 13.0%.

6. Implications and Conclusions

With this paper we aim at contributing to the field by modeling, quantifying, and testing sensitivities of the risks associated with private equity fund-of-fund investments. Firstly, we designed a detailed *cash flow model* which allows investors to assess the risk and return profile for private equity FoF investments. For this purpose we adopted and extended the VaR approach to the two most common measures of return, namely IRR and multiple. The risk measures introduced, iRaR and MaR, are conceptually capable of grasping non-negative risk positions, which is particularly important for the private equity asset class due to its successful historical performance. Further, although simplifying some aspects, we are convinced that our model represents a good compromise between manageability and complexity. Hence, the model might be used as a basis for the internal use of an investor's (customized) estimation of risk exposure, e.g. according to bank regulation guidelines (among these the German MaRisk guidelines, for example, regulate the internal risk management of financial institutions). Further, the structured modeling creates transparency for investors as well as FoF managers, e.g. for future negotiations of discretionary terms.

Our *quantification* on a proprietary sample might serve as a starting point for such considerations. On a general level, our analysis has shown that investing in FoF instead of direct investments in private equity funds really offers risk diversification gains. Our results for the risk measures iRaR and MaR (15.32% or 0.54x) for the entire asset class show that there is a significant variation in returns that put investors at a risk of not fully participating in the potential of the asset class. However, in our sample the value at risk, in the original sense, is never zero, irrespective of any common confidence level chosen.

More detailed, our *sensitivity analyses* revealed that the FoF's strategy, e.g. in terms of investment ratio and management fee, has a strong impact on risk and return. The first issue, in particular, is interesting, as we think it is typically not paid attention to in the screening of, or negotiation with, FoF or PEF. Our model shows that, based on historical transactions, the current discussions of reducing the management fee and carried interest from 2% to 1% and from 20% to 10% respectively increases the mean IRR by 4.3 percentage points and the mean mDIC by 13.0%.

In addition, we hope that scholars in the field of private equity research might find some new aspects in our conceptual development of the FoF level that is typically not considered. This allows identifying risk and return contributions that can be gained by inserting an additional layer of diversification. Further, our explicit treatment of the FoF level and the results of our empirical work have shown that, while they share key features, significant differences exist between key drivers of FoF' and private equity funds' return and risk.

We hope that our work can inspire future research that overcomes the limitations of the present work. This does not only refer to the level of sophistication in terms of conceptually modeling such an investment cycle, but also to the assembly of appropriate samples. Given the bias in our sample, more representative and larger samples containing cash flow information at the portfolio company level are required to find more meaningful results for private equity as an asset class. This would also

allow investigating further aspects we deem to be of importance, such as introducing a time dimension in order to account for market cycles in the private equity industry. We have provided a first indication for the intuitively appealing argument that 'bad timing' in setting up a fund (on both levels) does make a difference.

Appendix A. Figure 10

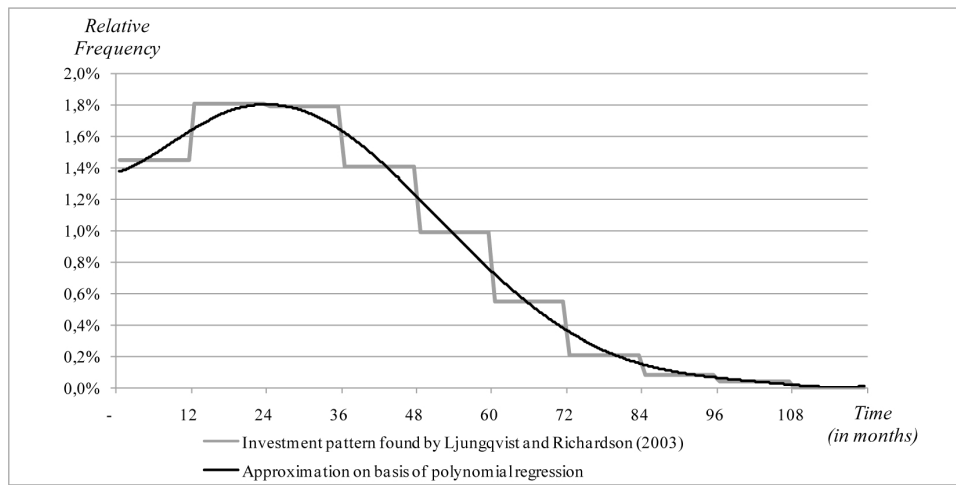


Fig. 10. Investment delays private equity investments.

Appendix B. Figure 11

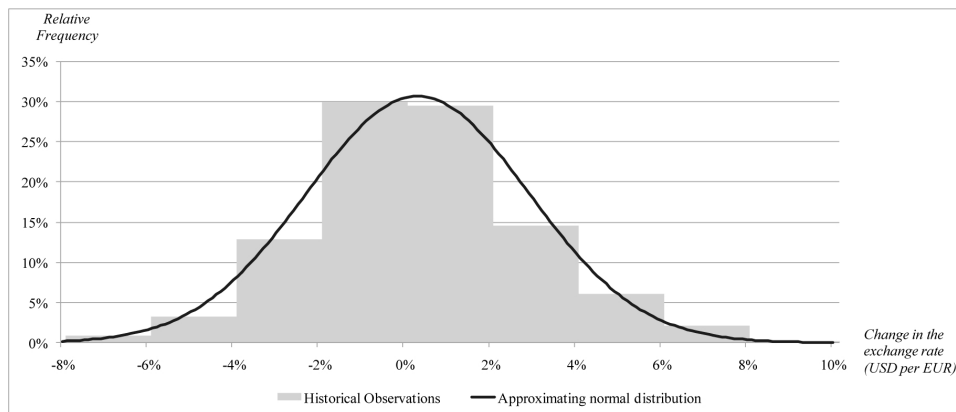


Fig. 11. Historical density function of the change in exchange rates and approximated by a normal distribution.

Appendix C. Figure 12

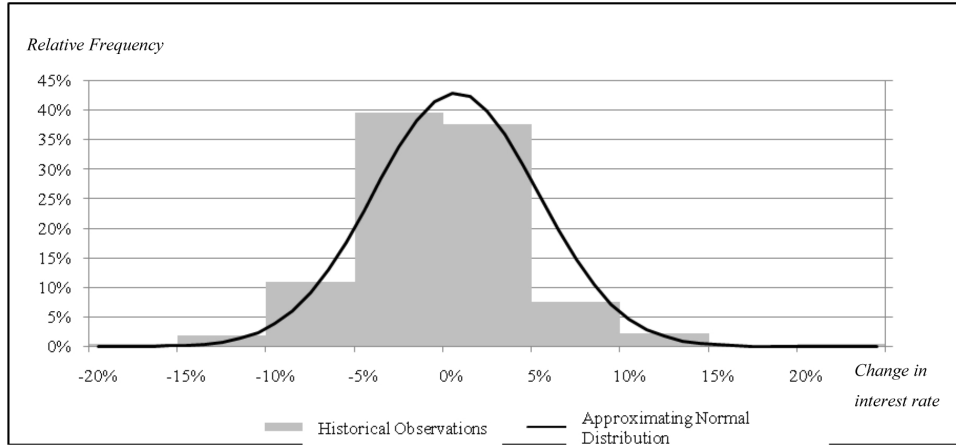


Fig. 12. Historical density function of the monthly change in the EURIBOR interest rate and approximated by a normal distribution.

Appendix D. Table 2

Table 2. Impact of changes in input parameters on mean and iRaR 95% based on IRR.

IRR	Decrease by 10%		Increase by 10%			
	Mean	iRaR 95%	Mean	iRaR 95%		
PEF Hurdle	-0,03%	-0,20%	0,05%	0,00%		
PEF Lifetime	1,50%	1,77%	-1,18%	-1,74%		
PEF IP	0,38%	0,59%	-0,36%	-0,59%		
PEF Carry	1,87%	1,77%	-1,81%	-2,13%		
PEF Tranches	0,37%	0,33%	-0,41%	-0,32%		
PEF Diversification	0,14%	-2,60%	-0,21%	0,91%		
PEF MF	2,10%	2,68%	-2,07%	-2,70%		
PEF IR*	-2,33%	-3,03%	1,35%	1,73%	0,58%	0,78%
FoF Hurdle	0,00%	0,00%	0,00%	0,00%		
FoF Lifetime	0,11%	0,03%	-0,03%	0,00%		
FoF IP	0,28%	0,43%	-0,27%	-0,52%		
FoF Carry	0,57%	0,67%	-0,57%	-0,62%		
FoF Tranches	0,71%	0,67%	-0,76%	-0,96%		
FoF Diversification	0,20%	-1,66%	0,11%	0,92%		
FoF MF	1,85%	2,34%	-1,87%	-2,38%		
FoF IR*	-2,93%	-3,47%	1,66%	1,91%	0,72%	0,73%

Appendix E. Table 3

Table 3. Impact of changes in input parameters on mean and MaR 95% based on DIC.

DIC	Decrease by 10%		Increase by 10%			
	Mean	MaR 95%	Mean	MaR 95%		
PEF Hurdle	-0,03%	-0,07%	0,05%	0,04%		
PEF Lifetime	0,68%	0,87%	-0,74%	-0,77%		
PEF IP	0,37%	0,38%	-0,37%	-0,36%		
PEF Carry	1,24%	1,03%	-1,22%	-1,04%		
PEF Tranches	-0,03%	0,00%	0,03%	0,04%		
PEF Diversification	0,15%	-0,63%	-0,14%	0,49%		
PEF MF	1,21%	1,14%	-1,17%	-1,11%		
PEF IR*	-1,32%	-1,27%	0,78%	0,77%	1,11%	1,08%
FoF Hurdle	0,00%	0,00%	0,00%	0,00%		
FoF Lifetime	0,19%	0,15%	-0,02%	-0,03%		
FoF IP	0,28%	0,28%	-0,28%	-0,28%		
FoF Carry	0,51%	0,40%	-0,50%	-0,39%		
FoF Tranches	-0,03%	-0,03%	0,05%	0,02%		
FoF Diversification	0,06%	-0,70%	0,04%	0,61%		
FoF MF	1,04%	0,99%	-1,02%	-1,01%		
FoF IR*	-1,10%	-1,08%	0,64%	0,61%	0,91%	0,86%

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