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Valuing a Private Equity Carried Interest as a Call Option on Fund Performance

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A private equity (PE) fund manager's carried interest is an important component of the fund manager's compensation. It is difficult to evaluate because it is an illiquid contingent claim whose value depends on the fund's expected future performance. We develop models for valuing the carried interest for three alternative structures and quantify the sensitivity of its value to the key value drivers. Our models should be of practical interest because the magnitude of PE fund manager compensation has come under scrutiny. Regulators have called on public pension funds that have invested heavily in PE funds to report all the fees they have paid (Martin [2015] and Gillers and Lim [2017]). Surprisingly, many large pension funds seem not to have done this calculation, which brings into question whether they might have unwittingly paid excessive fees. This question is economically important because PE fees typically absorb about 6% per year of the value invested (Phalippou and Gottschalg [2009]).

Our article contributes to the rich literature concerning PE fund manager compensation. Metrick and Yasuda [2010] provided a detailed analysis of PE fund manager compensation structures. PE fund manager compensation consists of a mixture of fees and a complex contingent carried interest in the fund, which represents a call

option on the fund's leveraged equity investment portfolio (Metrick [2007], Metrick and Yasuda [2010], and Sorensen, Wang, and Yang [2014]). Phalippou and Gottschalg [2009] and Metrick and Yasuda [2010] find that the carried interest has typically accounted for about one-third of the PE fund manager's compensation. At least some PE fund managers appear to possess scarce skills, which would enable the fund to generate positive alpha (α) (Metrick and Yasuda [2010] and Harris, Jenkinson, and Kaplan [2014]). Skilled managers presumably adjust the prices they charge for their services to capture as much of the rents from these skills as they can (Metrick and Yasuda [2010]).

We model a PE fund carried interest within a call option pricing framework. We model three alternative carried interest structures, which incorporate the investors' preferred return and the PE fund manager's carried interest catch-up. Our models express the value of the carried interest as the value of a single call option when there is no catch-up and as the difference between two call option values when there is a catch-up. Our models are similar to that of Sorensen, Wang, and Yang [2014] but differ in four key respects. First, Sorensen, Wang, and Yang [2014] assumed the PE fund purchases a portfolio of unlevered firms and borrows to finance these investments at the fund level; the entire debt must be repaid before the limited partners

can get any return on their investment. In our models, borrowing is done at the firm level rather than the fund level, and the PE fund manager manages a leveraged equity investment portfolio, which is the financing structure PE funds actually employ. Second, the carried interest call option in Sorensen, Wang, and Yang [2014] is written on the PE fund's portfolio of investments in unlevered firms. In our models, the call option is written on the fund's portfolio of investments in levered firms' equity. Third, our models explicitly incorporate the monitoring and transaction fees PE fund managers collect from their portfolio companies and the fraction of those fees they rebate to investors. Fourth, Sorensen, Wang, and Yang's model would be more difficult than ours to implement because theirs requires calculating the correlation matrix for the returns on a diversified portfolio of unlevered assets.

We quantify the sensitivity of the carried interest's value to the investors' preferred return, the investment portfolio's expected return and volatility, the fraction of monitoring and transaction fees rebated to investors, and the cost of the carried interest's illiquidity. We also calculate the carried interest delta for each structure to quantify the sensitivity of the value of the carried interest to increases in the value of the PE fund's portfolio and compare the incentive effects of the three carried interest structures. We also quantify how important the carried interest is to a PE fund manager's total compensation. Under reasonable assumptions, carried interest accounts for between one-third and one-half of fund manager expected total compensation. This conclusion bridges Phalippou and Gottschalg's [2009] and Metrick and Yasuda's [2010] empirical findings that the carried interest accounts for about one-third of the PE fund manager's compensation and Rouvinez's [2005] finding that the carried interest accounts for the majority of it.

Our study should be useful to PE fund investors who can negotiate fund manager compensation. Robinson and Sensoy [2013] found that during private equity boom periods, total PE fund manager compensation tends to increase, and this increase is driven by higher management fees and a shift in compensation away from variable compensation (carried interest) toward fixed compensation (fees). Our models will enable investors to value the carried interest and quantify expected total manager compensation, which will take on increased importance during periods when the mix

of compensation is shifting. These calculations can help investors negotiate a fund manager compensation structure that strikes an appropriate balance between annual fee and carried interest incentive compensation.

ECONOMIC CHARACTERISTICS OF PE FUND CARRIED INTEREST

PE funds are professionally managed pools of capital that invest in a portfolio of leveraged corporate equity. They are typically organized as limited partnerships. The general partner organizes the limited partnership, raises the capital, and manages the investment portfolio, and the limited partners provide the bulk of the investment capital.

PE Fund Manager Compensation

The PE fund manager selects, purchases, manages, and sells the investments on behalf of the fund. The investments usually phase in over a period of up to five years from the fund inception date (Stowell [2018]). The average length of the investing period is about three years, and the fund's investments are harvested between 6 and 10 years from the fund's inception date (Metrick and Yasuda [2010]).

The limited partners usually receive a preferred (or hurdle) rate of return on their contributed capital. They also receive an additional contingent return on their capital, which is typically about 80% of the fund's residual profit (Metrick and Yasuda [2010]). The preferred rate of return is generally between 7% and 10% per annum with 8% being most common (Rouvinez [2005]). The general partner receives annual management fees, which are typically about 2% of assets under management.¹ The general partner also receives a contingent carried interest, which is a residual claim on a PE fund's value after the fund pays management fees and other expenses, returns the limited partners' contributed capital, and pays their preferred rate of return. This contingent claim is designed to align the economic interests of the fund manager and investors. The percentage carried interest, referred to as the carry level, is typically about 20% (Rouvinez [2005], Metrick and Yasuda [2010], and Stowell [2018]).

PE fund managers usually charge each portfolio company a transaction fee based on the transaction's value each time the fund acquires a company and quarterly

monitoring fees while the fund owns the company (Phalippou, Rauch, and Umler [2016]). Monitoring fees are typically “accelerated” (and thus paid in full) if the fund sells the portfolio company or takes it public before the fee agreement expires. Phalippou, Rauch, and Umler [2016] reported that transaction fees amount to about 0.88% of the fund’s total enterprise value (TEV), monitoring fees amount to about 0.72% of TEV, and other fees (e.g., for arranging leveraged buy-out debt refinancing) represent about 0.15% of TEV. These fees are economically significant because they amount to nearly \$20 billion, or about 1.75% of the aggregate \$1.1 trillion TEV of the 592 LBOs in their sample. While the limited partners can negotiate the management fee and the carried interest, the PE fund manager solely determines the portfolio company fees when it enters into a management services agreement after it acquires a portfolio company. This negotiation is seldom at arm’s length because the PE fund controls the company, and Phalippou, Rauch, and Umler [2016] found that portfolio company fees are usually not based directly on the amount of work the fund manager performs. However, these fees are often at least partially rebated to fund investors by reducing the quarterly management fee by the rebated amount.² PE fund managers who share fees rebate on average about 50% of portfolio company fees (Phalippou, Rauch, and Umler [2016]). There has been mounting investor pressure on PE funds to increase the fraction they rebate.

PE Fund Carried Interest

Our models explicitly incorporate three of the four basic carried interest features that affect its value—portfolio expected rate of return and volatility, investors’ preferred rate of return, and catch-up provision—and implicitly take into account the fourth, a clawback feature.

Committed capital is the total amount of money the limited partners invest. Investment capital is this amount minus the cost of establishing the fund and the management fees. Net investment capital is this amount minus the fraction of monitoring, transaction, and any other fees that are not rebated to limited partners. The limited partners must recover their committed capital in full plus the promised hurdle rate of return on committed capital before the fund can make any carried interest distributions. This return of capital

includes repayment of the portion of their capital the fund spent on establishing the fund or paying management, monitoring, and transaction fees, and other fund expenses.

The catch-up provision allows a fund manager to receive a disproportionate share (and possibly all) of the fund’s profits once the fund generates sufficient profits to cover the investors’ hurdle rate of return until the manager’s profits distribution catches up with the investors’ profits interest.³ The fund manager catches up to the investors when its profit distribution reaches the agreed percentage (typically, 20%) of the total profits distributed (including the catch-up).

The clawback provision protects the limited partners if the fund loses money after the fund manager has started receiving profits distributions. This feature “claws back” distributions from the general partner in that case to ensure that the limited partners receive their promised preferred return and that the general partner does not receive more than the agreed carried interest percentage over the life of the fund. For simplicity, our models distribute profits to the fund manager at the end of the holding period after the fund has realized its total profit. The carried interest calculation takes into account the profits and losses on all the fund’s investments, which eliminates the need to perform a separate clawback calculation.

A carried interest has the payoff profile of a call option. The value of the call option derives from the appreciation in the value of the PE fund above the stated strike price before the call option expires. Likewise, the value of a PE carried interest is contingent on the portion of the fund’s profits that remain after the fund has returned the limited partners’ committed capital and paid their preferred return. The limited partners’ committed capital with the preferred rate of return accumulated to the terminal date is the carried interest option’s strike price. The option’s payoff will be zero if the terminal value of the portfolio is below the strike price. If the terminal value is above the strike price, the option payoff is equal to the general partner’s carried interest percentage of the difference between the portfolio’s terminal value and the strike price.

Example

Exhibit 1 illustrates the allocation of fund profits between limited partners and the general partner when

EXHIBIT 1

Distribution of Private Equity Fund Profits When There Is a Hurdle Rate of Return and a Catch-Up Feature

	Parameter	Value
Assumptions^[1]		
Net Investment Capital	$V_p(0)=H1$	\$100
Holding Period (years)	T	8
Preferred (Hurdle) Rate (per annum)	h	8%
Carry Level	c	20% ^[2]
Catch-up Rate	θ	100% ^[3]
Realized Investment Return (per annum)	r_e	15%
Distribution at the End of the Term		
Terminal Value of the Portfolio	$V_p(T)$	\$306 ^[4]
Preferred Return to Limited Partners	$H2$	\$164 ^[5]
Residual Profit		\$142 ^[6]
Carried Interest Catch-Up Proceeds		\$16 ^[7]
Allocation of the Residual Profit to Limited Partners		\$101 ^[8]
Allocation of the Residual Profit to General Partner		\$25 ^[9]
Value of the Carried Interest		\$41 ^[10]

^[1]This example assumes zero cost to establish and to operate the fund only for illustration purposes. Exhibit 4 provides a more detailed calculation that includes these costs.

^[2]Represents the percentage of residual profit of the fund distributed to the general partner. Consequently, 80% of the residual profit is distributed to the limited partners.

^[3]Represents the percentage of incremental profit of the fund distributed to the general partner during the catch-up period.

^[4] $= V_p(0) \times (1 + r_e)^T$

^[5] $= V_p(0) \times (1 + h \times T)$

^[6] $= V_p(T) - H2$

^[7] $= c \times (H2 - H1) / (1 - c)$

^[8] $= (1 - c) \times ([6] - [7])$

^[9] $= c \times ([6] - [7])$

^[10] $= [7] + [9]$

Notes: This exhibit illustrates how PE fund profits are distributed between the limited partners (the investors) and the general partner (fund manager) assuming a typical PE fund compensation structure that includes a preferred rate of return (hurdle rate) and a carried interest with a catch-up feature. The assumptions are provided in the upper panel, and the distribution of fund profits and fees is detailed in the lower panel.

there is hurdle rate of return and a catch-up. It includes the following variables:

- V_p : value of the PE fund's equity investment portfolio net of each portfolio company's indebtedness and any fees incurred.

- c : carry level, which represents the general partner's percentage carried interest.
- θ : catch-up rate, which is the portion of the fund's incremental profits distributed to the general partner during the catch-up period.
- h : hurdle rate of return on the portfolio, which according to industry practice, is usually expressed on a simple-interest basis.
- $H1$: the threshold amount of the terminal portfolio value for the carried interest when there is no hurdle rate of return.
- $H2$: the threshold amount of the terminal portfolio value for the carried interest when there is a hurdle rate of return.

To simplify the illustration, assume zero costs to establish and operate the fund. The initial committed capital is also the net investment capital. We assume that the initial value of the fund's portfolio is \$100, the holding period is eight years, the hurdle rate of return is 8% per annum, the carry level is 20%, the catch-up rate is 100%, and the realized annual investment return is 15%. The terminal value of the portfolio is \$306 ($= \$100 \times [1 + 15\%]^8$). The fund's total profit is \$206 ($= \$306 - \$100$). With an 8% hurdle rate of return, the limited partners receive \$164 ($= \$100 \times [1 + 8 \times 8\%]$). The residual profits are \$142 ($= \$306 - \$164$), which are allocated between the limited partners and the general partner in accordance with the carry level. Due to the catch-up feature, the fund manager's profit distributions will catch up to the investors' distributions when its profit distribution reaches 20% of the total profits distributed. The catch-up proceeds amount to \$16 ($= \frac{20\% \times [\$164 - \$100]}{1 - 20\%}$). The residual profits less the catch-up proceeds are \$126 ($= \$142 - \$16$), of which 80% (\$101) is allocated to the limited partners and 20% (\$25) is allocated to the general partner. The value of the carried interest equals \$41, which is the sum of the catch-up distribution (\$16) and the allocation of the residual profits to the general partner (\$25). Due to the catch-up feature, the fund manager receives 20% of the total profits ($\$206 \times 20\% = \$41$), and the investors get 80% ($\$206 \times 80\% = \$165$).

CARRIED INTEREST VALUATION MODELS

Valuers typically use two types of models to value PE carried interest: a discounted cash flow (DCF)

model (Zhou and Kam [2013]) and the conventional Black–Scholes–Merton (BSM) call option pricing model (OPM) (Fleischer [2005] and Anson [2012]). However, a DCF model cannot accurately capture the contingent nature of the carried interest. A single OPM can be used when the fund has a very simple structure that excludes any preferred return. However, it cannot accommodate the preferred return and catch-up features typically found in a PE carried interest. Moreover, the BSM OPM should not be used to value a PE carried interest because it assumes liquid markets for the option and the underlying asset, which a PE carried interest does not have, and it does not allow for the possibility that fund managers are able to achieve positive portfolio α due to their skill in choosing, managing, and harvesting the fund's investments. Nevertheless, we develop OPMs analogous to the BSM OPM for valuing PE carried interest under reasonable assumptions.

PE Carried Interest OPMs

In our PE carried interest OPMs, the carried interest is a single simple call option or a pair of simple call options, depending on whether the carried interest includes a preferred rate of return with a catch-up feature. We value each simple call option by applying a call OPM that is analogous to the BSM call OPM (Black and Scholes [1973] and Merton [1973]). Our models allow for a PE fund manager's special investment skill, which can produce positive portfolio α .

We make the same simplifying assumptions as Sorensen, Wang, and Yang [2014] and Bollen and Sensoy [2016] regarding investment timing. Investors fully commit their capital at time 0. The fund manager invests the entire fund, except for money set aside to pay future management, monitoring, and transaction fees, at a single point in time we call the weighted average time to invest. She harvests all the investments at a single later point in time we call the weighted average time to exit. Thus, the holding period is fixed whereas, in reality, it may be endogenously chosen by the fund manager. For example, Robinson and Sensoy [2013] and Gredil [2015] found that some PE fund managers tend to delay harvesting investments until the hurdle rate has been met and thereafter accelerate their harvesting, especially when the fund's carried interest includes a catch-up. They also found that managers of poorly performing funds tend to delay selling losers and taking losses to

sustain the flow of management fees. Fixing the holding period improves model tractability.

We develop our models within a fully risk-adjusted framework. We make the same hedging assumptions as Sorensen, Wang, and Yang [2014] in their full spanning case to derive the basic OPMs and then adjust for the illiquidity of the carried interest. That is, we initially assume that the leveraged equity portfolio risk, the general partner's carried interest risk, and the limited partners' investment risk are fully spanned in the public equity market. The general partner and the limited partners can perfectly hedge their leveraged equity portfolio risks, the general partner can perfectly hedge any incremental risk of holding the carried interest, and the limited partners can perfectly hedge any incremental risk of holding their limited partnership interests, in each case by dynamically adjusting their holdings of public equity securities and the risk-free asset. Accordingly, the fund manager and the limited partners are not exposed to any unhedged risk, the fund manager's compensation stream and the limited partners' expected returns can each be discounted at the risk-free rate, and there is no cost of illiquidity attached to the limited partners' investment.⁴

The value of a PE carried interest depends on eight parameters:

1. The initial value of the PE fund's levered equity investment portfolio (the spot price of the underlying asset),
2. The average length of the investment holding period (the time until the option expires),
3. The riskless rate for a maturity equal to the option's time to expiration,
4. Priority payment to the limited partners including their preferred return (the option strike price),
5. Any interim cash distributions (which we assume are zero),
6. The expected rate of return on the fund's levered equity investment portfolio,
7. The volatility of the return on the fund's levered equity investment portfolio, and
8. The discount for lack of marketability to capture the cost of illiquidity.

The volatility parameter σ is fixed. Because the volatility can change during the life of the fund, σ should be interpreted as the expected average volatility of the return on the fund's levered equity investment portfolio over the life of the fund.

PE funds offer investors the opportunity to realize a premium rate of return due to the fund manager's special industry knowledge and skill in choosing, managing, and harvesting the fund's investments (Metrick and Yasuda [2010]). PE fund managers in turn earn an economic rent on their intellectual capital through the carried interest. Empirical evidence indicates that while PE funds on average generate positive α even after fees, fund performance is highly variable (Metrick and Yasuda [2010] and Harris, Jenkinson, and Kaplan [2014]) and has trended downward in recent years (Fan, Fleming, and Warren [2013]). We model this premium in annual rate of return as a PE-fund-specific constant $\alpha > 0$ ($\alpha < 0$ would indicate unprofitable performance). Following Goetzmann, Ingersoll, and Ross [2003], we model a PE fund's α as $(\mu - r_f - \beta[r_m - r_f])$ where μ is the expected rate of return of the portfolio, r_f is the risk-free rate, r_m is the market rate of return on a broadly diversified equity portfolio, and β is the portfolio's beta. We assume that prospective investors can estimate α based on the information the PE fund provides in its offering materials. The value of the PE fund's investment portfolio grows at an average annual rate $r_f + \alpha$ in the risk-adjusted framework.

Structure 1: No Hurdle Rate of Return

There are three alternative structures for the carried interest depending on whether there is a hurdle rate of return and a catch-up. Exhibit 2 provides payoff diagrams for the three structures. When there is no hurdle rate of return (and therefore no catch-up provision), the threshold amount at the time to exit, defined as $H1$, is equal to the amount of committed capital (CP). Therefore, before any adjustment for illiquidity, the value of a carried interest is the expected present value of the contingent profits interest:

$$\begin{aligned} & \text{Value of carried interest} \\ &= ce^{-r_f T} \int_{H1}^{\infty} [V_p(T) - H1] f(V_p) dV_p \end{aligned} \quad (1)$$

Evaluating Equation 1 leads to a pricing formula, which we denote CALL OPM, that is analogous to the BSM call OPM, with the following parameters: net investment capital at the inception of the fund ($V_p(0)$), volatility (σ) of the return on the fund's levered equity investment portfolio, rate of return ($r_f + \alpha$) of the fund's investment portfolio, committed capital ($H1$) as a strike price, and

the weighted average time to exit (T) for the investments in the fund's portfolio measured from the inception date of the fund. The terminal value of the portfolio $V_p(T)$ is the net investment capital at the inception of the fund compounded forward to the expiration date at the risk-adjusted expected rate of return of the investment portfolio. The fund rebates a fraction of the monitoring and transaction fees (possibly zero) to the limited partners, which effectively increases the amount of net investment capital the fund has to invest. The carried interest is discounted at the risk-free rate in the risk-adjusted setting.

The value of the carried interest is equal to the carry level c multiplied by the compound excess return per dollar invested $e^{\alpha T}$ and also multiplied by the CALL OPM value:⁵

$$\begin{aligned} & \text{Value of carried interest} \\ &= ce^{\alpha T} [V_p(0)N(d_1) - H1e^{-(r_f + \alpha)T}N(d_2)] \end{aligned} \quad (2A)$$

$$= ce^{\alpha T} \text{CALL} [V_p(0), H1, r_f + \alpha, \sigma, T] \quad (2B)$$

where $N(\cdot)$ is the standard normal pdf,

$$d_1 = \frac{\ln\left(\frac{V_p(0)}{H1}\right) + \left(r_f + \alpha + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

and

$$d_2 = \frac{\ln\left(\frac{V_p(0)}{H1}\right) + \left(r_f + \alpha - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (2C)$$

$\text{CALL} [V_p(0), H1, r_f + \alpha, \sigma, T]$ is the value of a call option on an asset whose initial value is $V_p(0)$, strike price is $H1$, interest rate is $r_f + \alpha$, volatility is σ , and time to expiration is T .

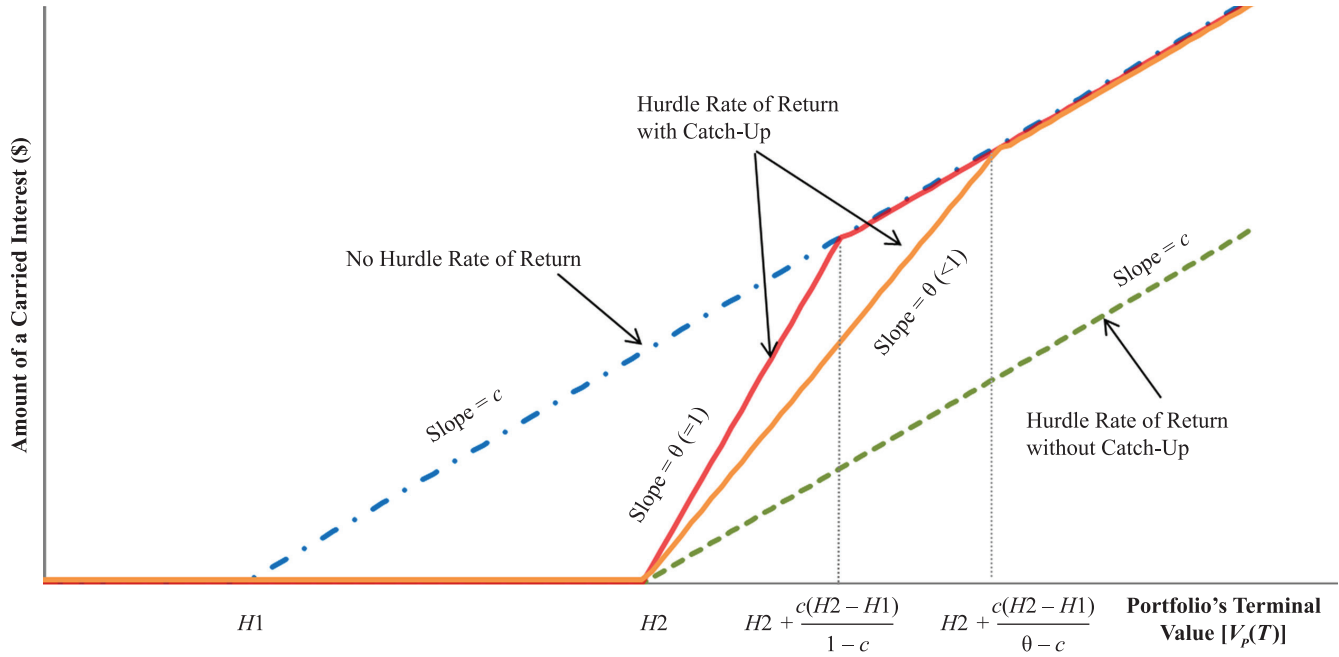
Structure 2: Hurdle Rate of Return without Catch-Up

When there is a hurdle rate of return but no catch-up, the threshold amount at the time to exit, defined as $H2$, is equal to $CP \times (1 + h \times T)$. The value of a carried interest is:

$$\begin{aligned} & \text{Value of carried interest} \\ &= ce^{-r_f T} \int_{H2}^{\infty} [V_p(T) - H2] f(V_p) dV_p \end{aligned} \quad (3)$$

EXHIBIT 2

Payoff Diagram for the Value of a Carried Interest



Notes: Payoff diagram for a carried interest expressed as a function of the portfolio's terminal value under three alternative structures: (i) no hurdle rate of return, (ii) hurdle rate of return without catch-up, and (iii) hurdle rate of return with catch-up. c is defined as the percentage carried interest. θ is defined as the catch-up rate. $H1$ and $H2$ represent the value of net investment capital and the value of the hurdle feature (the minimum asset value required to meet the preferred return at the time to exit), respectively.

Evaluate Equation 3 by applying CALL OPM with strike price $CP \times (1 + h \times T)$. The value of the carried interest is equal to the carry level c multiplied by the compound excess return per dollar invested $e^{\alpha T}$ and also multiplied by the CALL OPM value:

Value of carried interest

$$= ce^{\alpha T} [V_p(0)N(d_1) - H2e^{-(r_f + \alpha)T}N(d_2)] \quad (4A)$$

$$= ce^{\alpha T} \text{CALL}[V_p(0), H2, r_f + \alpha, \sigma, T] \quad (4B)$$

where $N(\cdot)$ is the standard normal pdf,

$$d_1 = \frac{\ln\left(\frac{V_p(0)}{H2}\right) + \left(r_f + \alpha + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

and

$$d_2 = \frac{\ln\left(\frac{V_p(0)}{H2}\right) + \left(r_f + \alpha - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (4C)$$

Structure 3: Hurdle Rate of Return with Catch-Up

When there is a hurdle rate of return and a catch-up, the threshold amount at the time to exit, $H2$, is $CP \times (1 + h \times T)$. The general partner gets a portion of the added return θ until the terminal portfolio value reaches $\left\{H2 + \frac{c(H2 - H1)}{\theta - c}\right\}$. When portfolio value reaches this threshold, the limited partners and the general partner share any further increase in value in accordance with the agreed carried level c .

The value of a carried interest is:

Value of carried interest

$$= \theta e^{-r_f T} \int_{H2}^{H2 + \frac{c(H2 - H1)}{\theta - c}} [V_p(T) - H2] f(V_p) dV_p + ce^{-r_f T} \int_{H2 + \frac{c(H2 - H1)}{\theta - c}}^{\infty} [V_p(T) - H1] f(V_p) dV_p \quad (5)$$

Equation 5 can be interpreted with the help of Exhibit 2. The first term is the value attributable to the catch-up period. The fund manager earns, as a carried interest, a portion θ of the increase in portfolio value between $H2$ and $\left\{H2 + \frac{c(H2-H1)}{\theta-c}\right\}$ to catch up to the limited partners after these investors have received their promised hurdle rate of return. The second term is the carried interest on any subsequent increase in portfolio value, which is equal to the fraction c of this increase.

Equation 5 is the composite of three call option positions:

a) $\theta e^{\alpha T} \times \text{Value of call option struck at } H2$ minus

b) $\theta e^{\alpha T} \times \text{Value of call option struck at}$

$$\left\{H2 + \frac{c(H2-H1)}{\theta-c}\right\} \text{ plus}$$

c) $c e^{\alpha T} \times \text{Value of call option struck at}$

$$\left\{H2 + \frac{c(H2-H1)}{\theta-c}\right\}.$$

The CALL OPM applies to each call option with the same parameters as Structure 1 except the strike price is either $H2$ or $\left\{H2 + \frac{c(H2-H1)}{\theta-c}\right\}$ depending on which option is being valued. Combining call option positions a), b), and c), the value of the carried interest is equal to the difference between the adjusted values of two call option positions:

Value of carried interest

$$\begin{aligned} &= \theta e^{\alpha T} \times \text{Value of call option struck at } H2 - (\theta-c)e^{\alpha T} \\ &\quad \times \text{Value of call option struck at } \left\{H2 + \frac{c(H2-H1)}{\theta-c}\right\} \\ &= \theta e^{\alpha T} \times (V_p(0)N(d_1) - H2e^{-(r_f+\alpha)T}N(d_2)) - (\theta-c)e^{\alpha T} \\ &\quad \times \left(V_p(0)N(d'_1) - \left(H2 + \frac{c(H2-H1)}{\theta-c}\right)e^{-(r_f+\alpha)T}N(d'_2)\right) \end{aligned} \quad (6A)$$

$$\begin{aligned} &= \theta e^{\alpha T} \text{CALL}(V_p(0), H2, r_f + \alpha, \sigma, T) - (\theta-c)e^{\alpha T} \\ &\quad \times \text{CALL}\left(V_p(0), H2 + \frac{c(H2-H1)}{\theta-c}, r_f + \alpha, \sigma, T\right) \end{aligned} \quad (6B)$$

where $N(\cdot)$ is the standard normal pdf,

$$d_1 = \frac{\ln\left(\frac{V_p(0)}{H2}\right) + \left(r_f + \alpha + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\ln\left(\frac{V_p(0)}{H2}\right) + \left(r_f + \alpha - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d'_1 = \frac{\ln\left(\frac{V_p(0)}{H2 + \frac{c(H2-H1)}{\theta-c}}\right) + \left(r_f + \alpha + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

and

$$d'_2 = \frac{\ln\left(\frac{V_p(0)}{H2 + \frac{c(H2-H1)}{\theta-c}}\right) + \left(r_f + \alpha - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (6C)$$

EXAMPLE

This section illustrates the valuation of a carried interest that has Structure 3. The PE fund will have a 10-year life: a 5-year investing period followed by a 5-year holding period. Based on data in Metrick and Yasuda [2010], we assume the fund invests 26%, 23%, 25%, 18%, and 8% of the portfolio in year 1 through year 5, respectively. The weighted average time to invest is approximately 2.09 years. Also based on Metrick and Yasuda [2010], we assume the time to exit follows an exponential distribution. Assuming a continuously compounded exit rate of 20% per year, the weighted average time to exit is 7.99 years from fund inception.

The net investment capital at the fund's inception is available to invest and serves as the spot price for the CALL OPM. It equals the committed capital *minus* the establishment cost *minus* also the present value of the management, monitoring, and transaction fees. Committed capital is \$100. Establishment cost is 1% of committed capital, or \$1. We assume a 2% annual management fee calculated on the amount of committed capital during the investing period and on the amount of net invested capital during the holding period. The

EXHIBIT 3

Investment Portfolio Expected Return and Volatility

Panel A: Investment Expected Return and Volatility

Investment (<i>i</i>)	1	2	3	4	5	6	7	8	9	10
Expected Return	10%	18%	26%	12%	15%	9%	25%	12%	6%	27%
Volatility (σ_i)	31%	49%	62%	49%	34%	27%	44%	24%	47%	46%
Weight (w_i)	10%	10%	10%	10%	10%	10%	10%	10%	10%	10%

Panel B: Investment Portfolio Expected Return and Volatility

Portfolio Expected Return (μ) ^[1]	16.00%
Portfolio Volatility (σ) ^[2]	18.51%

Panel C: Risk-Adjusted Expected Return of the Investment Portfolio^[3]

Risk-Free Rate (r_f)	2.00%
Supply-Side Risk Premium ($r_m - r_f$)	7.00%
Portfolio Beta (β)	1.00
Portfolio Alpha (α) ^[4]	7.00%
Risk-Adjusted Expected Return ($r_f + \alpha$)	<u>9.00%</u>

^[1]Expected average annual rate of return on fund equity net of monitoring fees. This calculation takes into account the average amount the fund invests in each portfolio company.

^[2]The volatility of the investment portfolio is calculated as the square root of the portfolio variance: $\sum_{i=1}^n \sum_{j=1}^n \rho_{ij} w_i w_j \sigma_i \sigma_j$.

^[3]The risk-adjusted expected return is calculated based on the following formula: $\alpha = \mu - r_f - \beta(r_m - r_f)$. See Goetzmann, Ingersoll and Ross (2003), p. 1690.

^[4]Before taking fees into account.

Notes: This exhibit furnishes details regarding the calculation of the expected return and volatility for each of 10 assumed PE fund investments, the calculation of the PE fund's investment portfolio expected return and volatility, and the calculation of the risk-adjusted expected return of the investment portfolio. The assumed investment portfolio expected return and volatility are incorporated into the example in Exhibit 4.

present value of the management fees at a 2% risk-free rate is \$13.55. The general partner often shares the monitoring and transaction fees with the limited partners (Phalippou, Rauch, and Umler [2016]). We assume initially that the general partner rebates 50% to the limited partners. Based on empirical data in Phalippou, Rauch, and Umler [2016], we assume the present value of the monitoring fees is 0.72% and the present value of the transaction fees is 0.88% of investment capital. Finally, the inception net investment capital is \$84.76.

The PE fund investment portfolio consists of 10 leveraged equity investments. We estimate the risk-adjusted expected rate of return and the volatility of the investment portfolio in Exhibit 3. We assume fund investors expect the fund manager to achieve investment portfolio $\alpha = 7\%$, which is generally consistent with empirical data. The risk-adjusted expected rate of return of the portfolio is 9%, the sum of the risk-free rate (2%) and portfolio α (7%). The portfolio volatility is 18.51%.

Exhibit 4 calculates the value of the carried interest applying the CALL OPM for Structure 3. The carry level is 20%, the catch-up rate is 80%, and the preferred rate of return is 8%. The spot price is \$84.76, the risk-adjusted expected rate of return of the portfolio from Exhibit 3 is 9% and the portfolio volatility is 18.51%, and the time to exit is 7.99 years. The strike price for the call option struck at H2 (Option 1) is \$163.90 ($= 100 \times [1 + 8\% \times 7.99]$), and the strike price for the call option struck at $\left\{ H2 + \frac{c(H2 - H1)}{\theta - c} \right\}$

(Option 2) is \$185.20. The value of the carried interest is $[\theta \times e^{\alpha T} \times \text{Value of Option 1}] \text{ minus } [(\theta - c) \times e^{\alpha T} \times \text{Value of Option 2}]$, or \$11.14 before any adjustment for illiquidity.

Finally, we value the fund manager's total compensation. Adding the present value of the management fees (\$9.42 for the investing period and \$4.13 for the holding period) and the present value of the monitoring and transaction fees (\$0.69) to the value of the carried

EXHIBIT 4

Valuing a Carried Interest Based on the Carried Interest Option Pricing Model

Assumptions	Parameter	Value	
Committed Capital	CP or $H1$	\$100.00	
Net Investment Capital at Inception of the Fund	$V_p(0)$	\$84.76 ^[1]	
Holding Period (years)	T	7.99	
Hurdle (Preferred) Rate (per annum)	h	8%	
Carry Level	c	20%	
Catch-Up Rate	θ	80%	

Valuation of a Carried Interest			
Parameters for the CALL OPM		Value for Option 1	Value for Option 2
Initial Value of Asset (Net Investment Capital)	S_0	\$84.76	\$84.76
Risk-Adjusted Expected Return of the Portfolio	$r_f + \alpha$	9.00%	9.00%
Volatility of the Portfolio	σ	18.51%	18.51%
Time to Exit	T	7.99	7.99
Dividend	d	0%	0%
Strike Price	K	\$163.90 ^[2]	\$185.20 ^[3]
Value of the CALL		\$19.54	\$15.44
Value of the Carried Interest			\$11.14 ^[4]

^[1]Equals committed capital less establishment costs (1%) less the present value of management fees (2% per annum) less monitoring fees (0.72% of investment capital) less transaction fees (0.88% of investment capital), assuming 50% rebates to the limited partners for monitoring fees and transaction fees.

^[2] $= CP \times (1 + h \times T)$

^[3] $= H2 + c \times (H2 - H1)/(\theta - c)$

^[4] $= [\theta \times e^{\alpha T} \times \text{Value of Option 1}] - [(\theta - c) \times e^{\alpha T} \times \text{Value of Option 2}]$

Notes: This exhibit furnishes an example showing how the carried interest option pricing model values a carried interest when there is a hurdle rate of return (8%) and a catch-up rate (80%). The carry level is 20%. The assumptions for the valuation are provided in the upper panel, and the valuation details are provided in the lower panel.

interest, the fund manager's present value total compensation as of the fund's inception is \$25.38. The carried interest accounts for 44% of fund manager total compensation before adjusting for the carried interest's lack of liquidity.

SENSITIVITY ANALYSIS

The value of a PE carried interest is sensitive to the limited partners' preferred return, investment portfolio expected return and volatility, the fraction of monitoring and transaction fees rebated to limited partners, and the cost of illiquidity.

Sensitivity to the Preferred Rate of Return

Exhibit 5 quantifies the sensitivity of the carried interest's value to the preferred return based on the

previous example. Panel A calculates the value of the carried interest, and Panel B expresses this value as a percentage of expected fund manager compensation.

Carried interest value decreases as the preferred return increases. The carried interest never accounts for more than one-half of the fund manager's total compensation. The value of the catch-up feature is evident. For example, with an 8% preferred return, including a catch-up feature (Structure 3) increases carried interest value (relative to Structure 2) by 63% from \$6.84 to \$11.14, which should stimulate the fund manager's incentive to increase portfolio α .

Sensitivity to the Portfolio Expected Return and Volatility

At least some PE fund managers appear to possess skills that enable them to generate positive α (Metrick

EXHIBIT 5

Sensitivity Analysis for the Value of a Carried Interest—Sensitivity to Preferred Rate of Return

	Preferred Rate of Return										
	0%	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
Panel A: Value of a Carried Interest											
Structure 1	\$13.46	—	—	—	—	—	—	—	—	—	—
Structure 2	\$13.46	\$12.42	\$11.44	\$10.53	\$9.67	\$8.88	\$8.14	\$7.46	\$6.84	\$6.26	\$5.73
Structure 3	\$13.46	\$13.42	\$13.30	\$13.09	\$12.81	\$12.46	\$12.06	\$11.62	\$11.14	\$10.64	\$10.13
Panel B: Value of a Carried Interest as a Percentage of Expected Total Compensation											
Structure 1	49%	—	—	—	—	—	—	—	—	—	—
Structure 2	49%	47%	45%	43%	40%	38%	36%	34%	32%	31%	29%
Structure 3	49%	49%	48%	48%	47%	47%	46%	45%	44%	43%	42%

Notes: This exhibit illustrates the sensitivity of the value of a PE fund carried interest to the preferred rate of return (hurdle rate). The assumptions are provided in Exhibit 4. The preferred rate of return varies between zero and 10%. The calculated value of the carried interest is provided for each of the three carried interest structures in Panel A and is used to calculate the value of the carried interest as a percentage of the fund manager's expected total compensation (including carried interest, management fees, monitoring fees, and transaction fees) in Panel B.

and Yasuda [2010]). PE funds usually report their rates of return net of fees and carried interest, which typically absorb about 6% per year of the value invested (Phalippou and Gottschalg [2009]). Fan, Fleming, and Warren [2013] and Harris, Jenkinson, and Kaplan [2014] found that PE buyout funds have generated positive α averaging between 3% and 6% per year net of fees and carried interest since the 1980s (implying 9% to 12% α before fees), whereas PE venture capital (VC) funds underperformed the public equity market by 4% per year after fees (implying 2% α before fees) on average between 1999 and 2011. These studies imply PE portfolio α between 2% and 12% per year before fees with a median value of 7%.

Exhibit 6 provides the sensitivity of the carried interest's value to the portfolio's risk-adjusted expected return. The value of the carried interest increases as the portfolio's expected return increases. With an 8% preferred return and catch-up, the carried interest accounts for about one-third to one-half of the fund manager's total compensation, if the portfolio's risk-adjusted expected return is between 7% and 11%. Exhibit 6 highlights the value of the catch-up feature. For example, when portfolio α is 7%, adding the catch-up feature (Structure 3) more than doubles the value of the carried interest (relative to Structure 2) from \$5.43 to \$11.14.

Exhibit 7 illustrates the sensitivity of the carried interest's value to portfolio volatility. The value of the carried interest increases as volatility increases, just as one would expect with a call option. Roughly

doubling the volatility to 35% from 18.51% raises fund manager compensation by about one-third and increases the carried interest's percentage of fund manager compensation to 52% from 44% for Structure 3.

Sensitivity to the Fraction of Fees Rebated

Exhibit 8 quantifies the sensitivity of the carried interest's value to the fraction of monitoring and transaction fees rebated to limited partners. Reducing the fraction rebated to limited partners from 100% to zero increases the carried interest's value by between 3% and 5%. The carried interest accounts for approximately one-half of the manager's compensation for Structure 1, one-third for Structure 2, and 44% for Structure 3 but with very little variation as the rebate fraction varies between zero and 100%.

Cost of Illiquidity

Equations 2, 4, and 6 provide an upper bound on the value of a PE fund carried interest because they do not adjust for the cost of the carried interest's illiquidity. This cost is due to the lack of marketability of the general partnership interest. Longstaff [1995] and Finnerty [2012] provided models for measuring the discount for lack of marketability (DLOM). Longstaff [1995] assumed that the holder of the restricted security has perfect market-timing ability, whereas Finnerty [2012] assumed just the opposite, that the holder lacks

EXHIBIT 6

Sensitivity Analysis for the Value of a Carried Interest—Sensitivity to Portfolio Alpha and Risk-Adjusted Expected Rate of Return on the Portfolio

	Portfolio Alpha (Risk-Adjusted Expected Rate of Return)										
	2% (4%)	3% (5%)	4% (6%)	5% (7%)	6% (8%)	7% (9%)	8% (10%)	9% (11%)	10% (12%)	11% (13%)	12% (14%)
Panel A: Value of a Carried Interest											
Structure 1	\$5.39	\$6.61	\$8.02	\$9.62	\$11.43	\$13.46	\$15.73	\$18.24	\$21.00	\$24.03	\$27.35
Structure 2	\$1.39	\$1.88	\$2.51	\$3.29	\$4.26	\$5.43	\$6.84	\$8.50	\$10.44	\$12.69	\$15.25
Structure 3	\$3.54	\$4.58	\$5.85	\$7.35	\$9.11	\$11.14	\$13.45	\$16.05	\$18.95	\$22.16	\$25.67
Panel B: Value of a Carried Interest as a Percentage of Fund Manager's Expected Total Compensation											
Structure 1	27%	32%	36%	40%	45%	49%	52%	56%	60%	63%	66%
Structure 2	9%	12%	15%	19%	23%	28%	32%	37%	42%	47%	52%
Structure 3	20%	24%	29%	34%	39%	44%	49%	53%	57%	61%	64%

Notes: This exhibit illustrates the sensitivity of the value of a PE fund carried interest to portfolio alpha and the risk-adjusted expected rate of return on the PE fund's investment portfolio. The assumptions are provided in Exhibit 4. Portfolio alpha varies between 2% per year and 12% per year, and portfolio risk-adjusted expected rate of return varies accordingly between 4% per year and 14% per year. The calculated value of the carried interest is provided for each of the three carried interest structures in Panel A and is used to calculate the value of the carried interest as a percentage of the fund manager's expected total compensation (including carried interest, management fees, monitoring fees, and transaction fees) in Panel B.

EXHIBIT 7

Sensitivity Analysis for the Value of a Carried Interest—Sensitivity to the Volatility of the Portfolio Returns

	Volatility of the Portfolio Returns										
	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%	55%
Panel A: Value of a Carried Interest											
Structure 1	\$12.61	\$12.66	\$13.03	\$13.68	\$14.50	\$15.41	\$16.36	\$17.32	\$18.27	\$19.20	\$20.10
Structure 2	\$2.62	\$4.16	\$5.74	\$7.30	\$8.83	\$10.33	\$11.79	\$13.20	\$14.55	\$15.85	\$17.08
Structure 3	\$7.68	\$8.96	\$10.24	\$11.51	\$12.76	\$13.98	\$15.16	\$16.31	\$17.40	\$18.45	\$19.45
Panel B: Value of a Carried Interest as a Percentage of Expected Total Compensation											
Structure 1	47%	47%	48%	49%	50%	52%	53%	55%	56%	57%	59%
Structure 2	16%	23%	29%	34%	38%	42%	45%	48%	51%	53%	55%
Structure 3	35%	39%	42%	45%	47%	50%	52%	53%	55%	56%	58%

Notes: This exhibit illustrates the sensitivity of the value of a PE fund carried interest to the volatility of the returns on the PE fund's levered equity investment portfolio. The assumptions are provided in Exhibit 4. The volatility of the investment portfolio returns varies between 5% and 55%. The calculated value of the carried interest is provided for each of the three carried interest structures in Panel A and is used to calculate the value of the carried interest as a percentage of the fund manager's expected total compensation (including carried interest, management fees, monitoring fees, and transaction fees) in Panel B.

any special market-timing ability. Gredil [2015] furnished empirical evidence that at least some PE fund managers appear to be better informed than the typical investor about certain publicly traded firms' valuations, which gives them some ability to time the market when harvesting the PE funds' investments. The appropriate DLOM for any particular PE fund carried interest will depend on the fund manager's market-timing ability, but in any case, it will lie between the extremes provided by

the lookback put option and average-strike put option DLOM models.

Exhibit 9 considers three portfolio volatilities—10%, 18.51% (the value for σ calculated in Exhibit 3), and 30%—and adjusts the carried interest value by subtracting the cost of illiquidity for a range of DLOMs based on the average-strike put option (Finnerty [2012]) and lookback put option (Longstaff [1995]) DLOM models. We value the carried interest at the assumed

EXHIBIT 8

Sensitivity Analysis for the Value of a Carried Interest—Sensitivity to the Fraction of Fees Rebated to Limited Partners

Value of a Carried Interest

	Fraction of Monitoring Fees and Transaction Fees Rebated to Limited Partners										
	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Structure 1	\$13.68	\$13.64	\$13.59	\$13.55	\$13.51	\$13.46	\$13.42	\$13.38	\$13.33	\$13.29	\$13.25
Structure 2	\$6.99	\$6.96	\$6.93	\$6.90	\$6.87	\$6.84	\$6.81	\$6.78	\$6.74	\$6.71	\$6.68
Structure 3	\$11.36	\$11.31	\$11.27	\$11.23	\$11.18	\$11.14	\$11.10	\$11.05	\$11.01	\$10.96	\$10.92

Notes: This exhibit illustrates the sensitivity of the value of a PE fund's carried interest to the fraction of monitoring fees and transaction fees rebated to the PE fund's limited partners. The assumptions are provided in Exhibit 4. The fraction of fees rebated to limited partners varies between zero and 100%.

volatility, then use the same volatility to calculate a range of possible DLOMs, and finally, adjust the calculated carried interest value by multiplying by one minus the DLOM.⁶ The illiquidity-cost-adjusted value of the carried interest never accounts for a majority of fund manager compensation. It still accounts for between one-third and one-half of the fund manager's expected total compensation for Structure 1 and Structure 3, but much less for Structure 2.

The cost of illiquidity can reverse the sensitivity of the carried interest's value to an increase in portfolio volatility. Higher portfolio volatility increases the carried interest's option value in Exhibit 7 but also increases the DLOM, which can more than offset the increase in option value. For example, at the high end of the DLOM range in Exhibit 9, the illiquidity-cost-adjusted carried interest value decreases as portfolio volatility increases both in amount (from \$10.02 at 10% volatility to \$7.91 at 18.51% volatility to \$4.21 at 30% volatility for Structure 1) and as a percentage of fund manager compensation (from 41% to 36% to 23%). The impact would be most severe for skilled market-timers who should have the highest DLOMs and are likely to have the strongest financial incentive to try to substitute fee compensation for the (relatively less valuable) carried interest.

Carried Interest Delta

A carried interest is designed to align the economic interests of the fund manager and the limited partners by motivating the fund manager to maximize fund value (Metrick and Yasuda [2010]). To examine the strength of this incentive, we calculate each structure's carried interest delta (Δ), which is the rate of change in the value

of the carried interest (CI) with respect to a change in fund portfolio value (V_p), $\partial CI / \partial V_p$. Δ proxies for the incentive effect of the carried interest.⁷

The deltas for the three carried interest structures are:⁸

$$\Delta_1 = ce^{\alpha T} N(d_1), d_1 \text{ given by Equation 2C} \quad (7)$$

$$\Delta_2 = ce^{\alpha T} N(d_1), d_1 \text{ given by Equation 4C} \quad (8)$$

$$\Delta_3 = \theta e^{\alpha T} N(d_1) - (\theta - c)e^{\alpha T} N(d'_1), \\ d_1 \text{ and } d'_1 \text{ given by Equation 6C} \quad (9)$$

All three Δ s are monotonically increasing in α , which should incentivize a fund manager to strive for greater α .

Exhibit 10 and Exhibit 11 plot the carried interest Δ and value, respectively, as functions of the fund's portfolio value for the three structures for different catch-up rates.⁹ We calculate the carried interest Δ s and values as of the date the fund is fully invested, which is 2.09 years from the fund's inception and 5.9 (= 7.99–2.09) years before the call option is set to expire. The other parameter values are the same as in the previous example.

The carried interest Δ for each structure in Exhibit 10 is always positive, which should incentivize the PE fund manager to increase portfolio α . For Structures 1 and 2, Δ is a monotonically increasing function of portfolio value, which should reinforce this incentive. Δ for Structure 3, which is initially monotonically increasing, accelerates as the catch-up feature kicks in, again reinforcing the incentive to increase α . The Δ curve for Structure 3 in Exhibit 10 can explain Robinson and Sensoy's [2013] and Gredil's [2015] finding that some

EXHIBIT 9

Sensitivity Analysis for the Value of a Carried Interest—Sensitivity to the Cost of Illiquidity

Portfolio Volatility = 10%									
DLOM	6% ^[1]	8%	10%	12%	14%	16%	18%	20%	21% ^[2]
Value of a Carried Interest									
Structure 1	\$11.96	\$11.71	\$11.45	\$11.20	\$10.95	\$10.69	\$10.44	\$10.19	\$10.02
Structure 2	\$3.93	\$3.85	\$3.76	\$3.68	\$3.60	\$3.52	\$3.43	\$3.35	\$3.29
Structure 3	\$8.46	\$8.28	\$8.10	\$7.92	\$7.74	\$7.56	\$7.39	\$7.21	\$7.09
Value of a Carried Interest as a Percentage of Expected Total Compensation									
Structure 1	46%	45%	45%	44%	43%	43%	42%	42%	41%
Structure 2	22%	21%	21%	21%	20%	20%	19%	19%	19%
Structure 3	37%	37%	36%	36%	35%	35%	34%	34%	33%
Portfolio Volatility = 18.51%									
DLOM	10% ^[1]	14%	18%	22%	26%	30%	34%	38%	41% ^[2]
Value of a Carried Interest									
Structure 1	\$12.10	\$11.56	\$11.02	\$10.48	\$9.94	\$9.40	\$8.86	\$8.33	\$7.91
Structure 2	\$6.14	\$5.87	\$5.60	\$5.32	\$5.05	\$4.78	\$4.50	\$4.23	\$4.02
Structure 3	\$10.01	\$9.56	\$9.12	\$8.67	\$8.23	\$7.78	\$7.33	\$6.89	\$6.55
Value of a Carried Interest as a Percentage of Expected Total Compensation									
Structure 1	46%	45%	44%	42%	41%	40%	38%	37%	36%
Structure 2	30%	29%	28%	27%	26%	25%	24%	23%	22%
Structure 3	41%	40%	39%	38%	37%	35%	34%	33%	32%
Portfolio Volatility = 30%									
DLOM	16% ^[1]	23%	30%	37%	44%	51%	58%	65%	73% ^[2]
Value of a Carried Interest									
Structure 1	\$12.96	\$11.88	\$10.80	\$9.72	\$8.64	\$7.56	\$6.48	\$5.40	\$4.21
Structure 2	\$8.69	\$7.96	\$7.24	\$6.52	\$5.79	\$5.07	\$4.35	\$3.62	\$2.82
Structure 3	\$11.75	\$10.77	\$9.80	\$8.82	\$7.84	\$6.86	\$5.88	\$4.90	\$3.82
Value of a Carried Interest as a Percentage of Expected Total Compensation									
Structure 1	48%	45%	43%	41%	38%	35%	31%	28%	23%
Structure 2	38%	36%	34%	31%	29%	26%	23%	20%	17%
Structure 3	45%	43%	41%	38%	36%	33%	29%	26%	21%

^[1]Calculated from the average-strike put option DLOM.

^[2]Calculated from the lookback put option DLOM.

Notes: This exhibit quantifies the impact of the cost of illiquidity on the value of a PE fund carried interest. The assumptions are provided in Exhibit 4. The DLOM is calculated according to the lookback put option (Longstaff [1995]) and average-strike put option (Finnerty [2012]) models assuming the expected average volatility of the value of the PE fund's levered equity investment portfolio over the life of the fund is alternatively 10%, 18.51%, and 30%. The value of the carried interest and its percentage of the fund manager's expected total compensation (including carried interest, management fees, monitoring fees, and transaction fees) are calculated for each of the three carried interest structures.

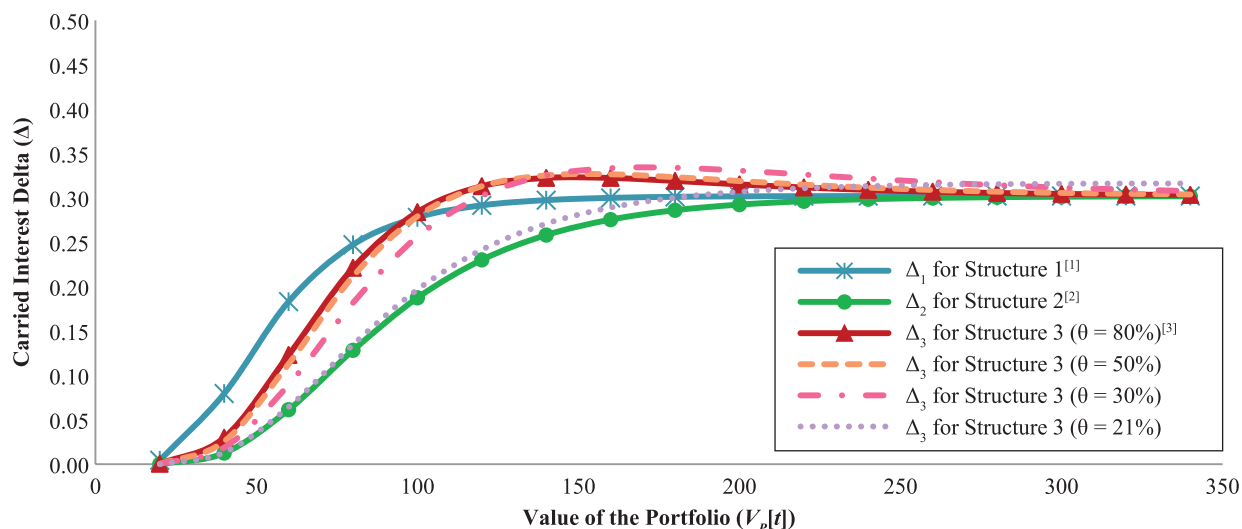
PE fund managers tend to delay harvesting investments before the fund achieves the preferred return (period of relatively low Δ) but accelerate their harvesting once it has met this threshold (period of relatively high Δ), especially when the fund's carried interest includes a catch-up. At asset values above $H2$, Structure 3 will

exhibit the highest Δ because of the catch-up feature's accelerated effect on the carried interest's value.

The carried interest's value increases monotonically with portfolio value for each structure in Exhibit 11. Carried interest value is always greater under Structure 1 than Structure 2 because Structure 1 has no preferred return, and the value under Structure 3 always lies in

EXHIBIT 10

Carried Interest Deltas—Sensitivity to the Value of the Portfolio



^[1] $\Delta_1 = ce^{aT} N(d_1)$, where d_1 is given by Equation 2C.

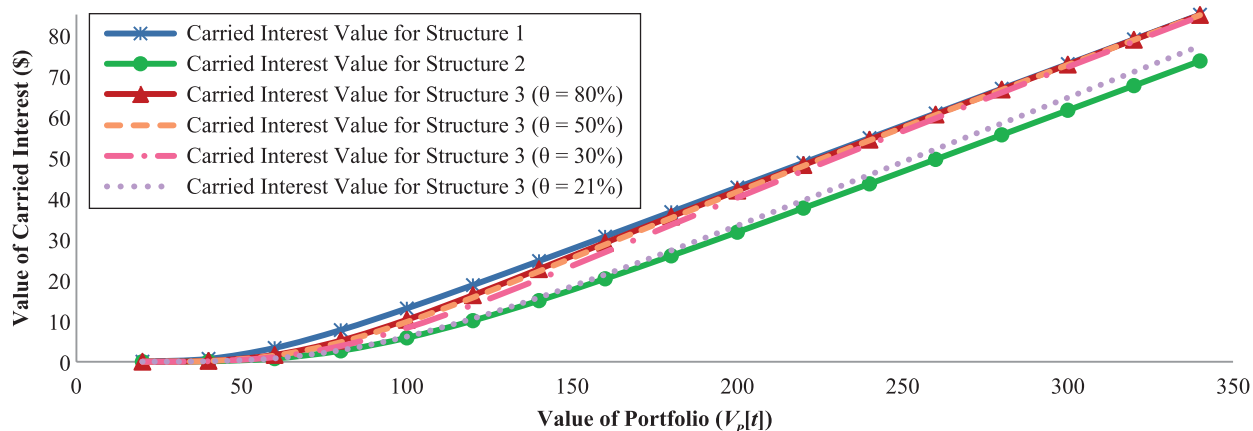
^[2] $\Delta_2 = ce^{aT} N(d_1)$, where d_1 is given by Equation 4C.

^[3] $\Delta_3 = \theta e^{aT} N(d_1) - (\theta - c)e^{aT} N(d_1')$, where d_1 and d_1' are given by Equation 6C.

Notes: This exhibit plots the carried interest delta as a function of the value of the PE fund's portfolio for three alternative carried interest structures. It is assumed that the fund is fully invested at a weighted average time to invest 2.09 years from the inception of the fund when the delta is calculated. It is also assumed that the weighted average time to exit from the portfolio investments is 7.99 years from the inception of the fund. The other parameter values are the same as in Exhibit 4.

EXHIBIT 11

Value of a Carried Interest—Sensitivity to the Value of the Portfolio



Notes: This exhibit plots the value of a carried interest as a function of the value of the PE fund's portfolio for three alternative carried interest structures. It is assumed that the fund is fully invested at a weighted average time to invest 2.09 years from the inception of the fund when the value is calculated. It is also assumed that the weighted average time to exit from the portfolio investments is 7.99 years from the inception of the fund. The other parameter values are the same as in Exhibit 4.

between, which is consistent with the option payoff diagram in Exhibit 2. Structure 2 is more favorable to limited partners than Structure 1 or Structure 3, and Structure 1 is least favorable. Structure 1 (Structure 2) is most (least) favorable to a fund manager. Thus, Structure 3, which is the most common (Metrick and Yasuda [2010]), appears to strike a compromise by giving investors a preferred return, which reduces their risk and incentivizes them to invest, and by giving a fund manager a catch-up, which motivates her to increase α and increases her compensation for doing so. Structure 3 has the added advantage that the catch-up feature gives a fund manager a stronger financial incentive to increase portfolio α .

CONCLUSIONS

PE fund managers receive a valuable call option on the fund's leveraged equity investment portfolio, which gives them a share of the fund's profits. We developed models to value the PE carried interest and quantified the sensitivity of this value to the key value drivers. The carried interest typically accounts for approximately one-third to one-half of fund manager compensation. The PE structure with preferred return and catch-up seems to strike a compromise that incentivizes limited partners to invest and fund managers to increase portfolio α , and the catch-up gives the fund manager a stronger financial incentive to increase portfolio α .

The models should be useful to PE investors who can negotiate manager compensation by helping them determine compensation arrangements that can strike the right balance between annual compensation and the carried interest.

ENDNOTES

¹Larger funds often agree to smaller management fees, sometimes as low as 1.5% of assets under management (Metrick and Yasuda [2010]).

²If the rebate amount would exceed the management fee for the quarter, the excess is typically carried forward to the next quarter.

³During the catch-up period, the general partner usually receives 100% of the incremental profits, although an 80/20 (general partner/limited partners) split, or something in between, is sometimes negotiated (Fleischer [2005]).

⁴The assumption of full spanning is weaker than the usual assumption of complete capital markets. With full

spanning, the risks of the PE investments are traded in the public equity market, but the PE fund can still earn positive α on its investment portfolio. Under the stronger assumption of complete markets, all positive α would be arbitrated away. This does not happen because the general partner's skill generates positive α , which the limited partners can share in only by investing in the PE fund and bearing the associated costs.

⁵Deriving the formulas for the carried interest under the full spanning assumption requires two adjustments to the standard BSM OPM share price Brownian motion process. First, the risk-adjusted drift rate is the risk-free rate plus the investment alpha, $r_f + \alpha$, rather than r_f . Second, the new risk-adjusted Brownian motion process fully captures all the systematic risk of the underlying leveraged equity investment returns, which enables us to discount the call option payoffs at the risk-free rate. A technical appendix furnishing the derivations of Equations 2, 4, and 6 is available from the authors.

⁶Because the management, monitoring, and transaction fees the fund manager receives are paid in cash and are assumed in our models to be set aside when the fund receives the investors' capital contributions, they bear no illiquidity cost.

⁷Specifically, the carried interest delta is $\partial CI / \partial V_p(0)$, where CI is the value of the carried interest in Equation 2, 4, or 6 for Structure 1, 2, or 3, respectively, and $V_p(0)$ is the value of the PE fund at the valuation date.

⁸The carried interest valuation models incorporate CALL OPM, which is analogous to the BSM call OPM, for which there is a very simple formula for the option delta (Hull [2012, pp. 380–382]).

⁹For illustrative purposes, we include $\theta = 21\%$ because θ should be greater than c (20% in the example) in Equation 6A.

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